MTH 201 - Mathematical Method 1 (Week 1-7) By Nuelgeek

<u>Week 1</u>

Real-valued functions of a single variable form the foundation of calculus and are crucial in science and engineering. These functions associate each input value with a single real number output, making them fundamental in mathematics and its applications. Understanding these functions enhances problem-solving skills and scientific inquiry, with particular relevance in economic analysis, where mathematical models are often expressed as functions.

Few attributes to take note of:

- Domain - All sets of possible inputs
- Range - All sets of actual output of a given input in a function, and this is the subset of co-domain
- Co-Domain- All sets of possible outputs
- Function Evaluation - Passing an actual value for x to get a corresponding output for the function
- Symmetry - Any shape that when folded it is equal
- Graph Transformation - the process by which an existing graph, or graphed equation, is modified to produce a variation of the preceding graph
- Inverse function - This type of function undoes whatever the original function does.

The lesson covered various ways to represent functions, including graphs, tables, and algebraic expressions. It then expanded to functions of several variables, explaining how to evaluate these functions at specific points and determine their domains and ranges. The concept of graphing functions of several variables was introduced, showing how these create surfaces in three-dimensional space. Level curves were also discussed as a method to visualize 3D functions on a 2D plane, similar to contour lines on a map. These tools and concepts provide powerful means to analyze and understand complex relationships in mathematics and real-world applications.

Coffee

Extra resources
Understanding Symmetry (Perfect and odd symmetry) -
Reflectional Symmetry and Rotational Symmetry | Don't Memorise
Logarithmic differentiation: Introduction to Logarithmic Differentiation (note: if your calculus foundation is shaking, you want to fix this especially because of this semester)
Transformations: Transformations of Functions | Precalculus

Week2

Logarithmic differentiation is a technique for differentiating complex functions, especially those with exponents or multiple factors. It's useful for functions raised to another function and products of several functions.
Steps for logarithmic differentiation:

Take the natural logarithm of both sides
Apply logarithm laws to simplify
Differentiate both sides
Solve for the derivative of the original function

This method simplifies products into sums and simplifies exponents.

Leibnitz's theorem provides a formula for the nth derivative of a product using binomial coefficients and derivatives of individual functions.
These techniques are valuable in advanced calculus, engineering, and physics, offering tools to solve complex mathematical problems and enhance problem-solving skills in various scenarios.

I also noticed the answers provided to some of the quizzes are wrong, so always find a way to cross check and compare your answers.

Week 3

We covered The Taylor's and Macluarin series and for this letter, we will call it "TMS", so what are these concepts all about? First, if you remembered calculus from last semester, you would notice that you have to solve a long lines of equations to get derivatives of functions and now imagine solving this for complex functions, the basic method of obtaining derivatives in calculus falls short when faced with complex functions, this is where we meet TMS.

TMS achieves this by approximating these complex functions as polynomials, which we now get to differentiate (calculus differentiation). When you get to differentiate now converted polynomial functions, we can easily find any higher order of the function's derivatives instead of going through long lines of equation to find for e.g the 30th derivative of ln(x), what do you say? What to take the challenge? LOL. But with TMS generated formula you can solve this in less than 3 minutes, but you have to first generate a general formula for the given function.

The difference between Taylor's theorem and Macluarin's is that with Taylor's theorem, you solve for the function at the center point and for Mac, you solve for the function at point zero.

Also, I noticed for all this to be all possible, the function itself should have unlimited order of differentiation(C-infinity) and derivatives that type of function is known as analytic function.

You can't use this formula to solve for something like f''''''(4x) (find the 6th derivatives), because if you do, in less than 5 steps you will get 0 as the last number which can't be differentiated.

Understanding Taylor's Theorem and macluarin series with its formula.

Oxford Calculus: Taylor's Theorem Explained with Examples and Derivat…

Taylor Series and Maclaurin Series - Calculus 2

Differentiation:

01 - Basic Derivatives in Calculus, Part 1 - Learn what a Derivative is an…

Week 4

Today we are covering a straight forward concept Rolle's Theorem, this is a mathematical concept that helps us learn about the behavior of functions within a closed interval [a, b] i.e. from one point to another. Before we can make use of this theorem on a function, Rolle requires that the function satisfies these 3 criteria:

- The function must be continuous in a closed interval [a, b], in graph terms this means the line must be continuous without breaks, jumps, or holes in the graph of f(x) on the interval, x-axis. You can draw the curve from f(a) to f(b) without lifting your pencil.
- f(a) must be equal to f(b)
- The function at point (a, b) must be differentiable, means the function is smooth, with no sharp corners or cusps in the interval, and the derivative exists everywhere inside the interval.

To solve or evaluate a function with Rolle's Theorem, follow these simple 3 steps

Step 1: Confirm f(a) = f(b)

Check if the function *f* has equal values at the endpoints of the interval [a, b], meaning f(a) = f(b). This is a critical condition for Rolle's Theorem to apply.

Step 2: Differentiate f(x)

Step 3: Solve for c within the interval [a, b]

When you find c, make sure it's first derivative f'(c) is equal to 0, what is "C"?

C in this Theorem is the slope of the tangent line at the curve, "$\cap$" i.e. the area where you draw a parallel line at the apex of the curve, and it just touches one point, will add an image.

The Mean Value Theorem (MVT), this theorem is used to determine that c exists between the interval [a, b], using the formula f'(c) = (f(b) - f(a)) / b - a just like your slope Y2 - Y1/X2 - X1.

So before you can use this formula to determine that c is correct or exist, you have to meet those 3 criteria above.

Steps to solving a problem

Step1: Find the f'(c)

Step2: use the MVT formula to find confirm f'(c) equals (f(b) - f(a)) / b - a

Extra resources:
Understanding Rolle's Theorem with examples : Rolle's Theorem
Understanding Mean Value Theorem Mean Value Theorem

Week 5

Recap of Basic Integration

Integration is essentially the reverse of differentiation. When we integrate a function, we find an expression whose derivative matches the original function.

∫ this sign is an integral or integration of a value

dx means we are doing something (integration in this case) with respect to x

X is the function or value we are integrating (as in integral)

Formula for integration: $\int x \, dx = (x^{n+1} / n + 1) + C$

Example: $\int x \, dx = (x^{1+1} / 1+1) + C$ ⇉ answer: $x^2/2 + C$

C is constant.

Introduction to Integration by Parts

Integration by parts is a technique used to integrate products of functions, and it's based on the product rule for differentiation.

Formula for Integration by Parts:

$\int u\,dv = uv - \int v\,du$

where:

- u and dv are frist & second parts of the function we're integrating.
- du is the derivative of u, and v is the integral of dv.

Why do we use this formula? Sometimes, when we have two functions multiplied together, it's difficult to integrate them directly. By choosing parts u and dv, we can simplify the integral by breaking it down.

How to Choose u and dv

There's a useful acronym to help choose u and dv: **LIATE**

- **L**ogarithmic (like ln(x)
- **I**nverse trigonometric (like arctan(x))
- **A**lgebraic (like x^2 or x)
- **T**rigonometric (like sin(x) or cos(x))
- **E**xponential (like e^x)

Generally, choose u from the first function you find on this list, and let the remaining part be dv.

Generally integration by part is actually easy, for "Reduction" you need to be more carefully.

So let's solve a question: Find the integral $\int xe^{x}\,dx$, i will add an image containing the solution.

A **reduction formula** is a way to simplify an integral by breaking it into a simpler one. The idea is to reduce the **power** or **complexity** of the integrand (the function you're integrating) in order to make the integral easier to solve.

Basic Idea of Reduction

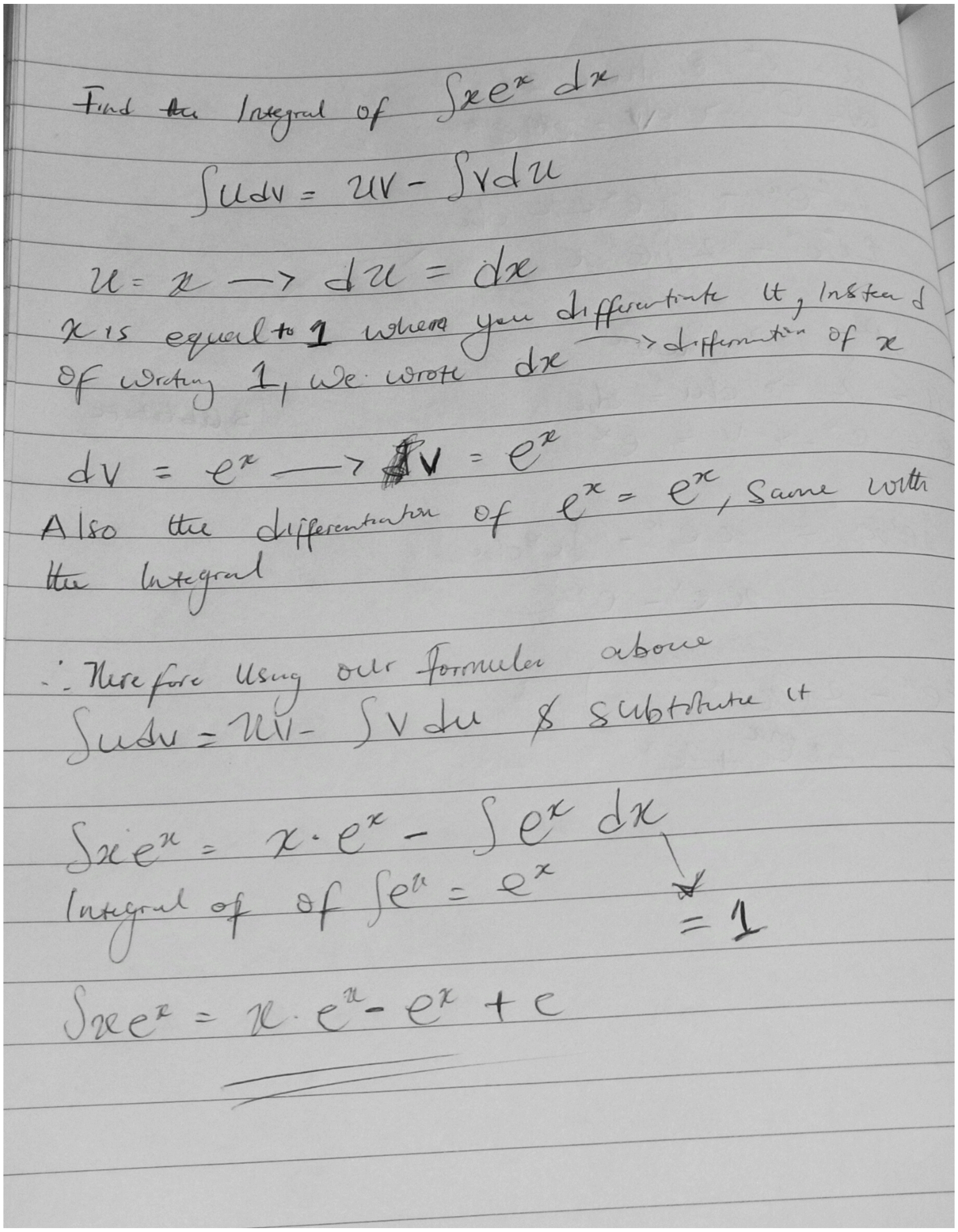
When you have an integral with a higher power of x (for example, x^{n}), the goal of a reduction formula is to find a way to express that integral in terms of a similar but **simpler** integral (with a smaller power of x).

The whole idea of reduction formula is find a way of reducing the right-hand side of $\int udv = uv - \int vdu$, this part ($\int vdu$) till we can now use the general formular of Reduction which $x^{n} e^{x} - n \int x^{n} e^{x}\,dx$,

First look at $x^{n} e^{x}$ as uv and $\int x^{n} e^{x}\,dx$ as $\int vdu$ and $-n$ as constant or helper to reduce the integral, So why all this?

Because was function is given to be integrated and it is in the form of product i.e a x b or ab or uv you will have to apply integration by part on the right-hand side of $\int v \times du$ which is also $\int vdu$ till there is no product form remaining, kind of a loop and this loop is based of the power of X^{n}, if n = 3, you will have use the integration by parts for $\int vdu$ for n = 3, 2, 1 till n = 0, hence we use

$x^{n} e^{x} - n \int x^{n} e^{x}\,dx$ as the general formula.


Find the Integral of $\int xe^x \, dx$

$$\int u\,dv = uv - \int v\,du$$

$u = x \longrightarrow du = dx$

x is equal to 1 where you differentiate it, Instead of writing 1, we wrote dx → differentiation of x

$dv = e^x \longrightarrow v = e^x$

Also the differentiation of $e^x = e^x$, Same with the Integral

∴ Therefore Using our formula above

$\int u\,dv = uv - \int v\,du$ & substitute it

$\int xe^x = x \cdot e^x - \int e^x \, dx$

Integral of of $\int e^x = e^x$ ($dx = 1$)

$\int xe^x = x \cdot e^x - e^x + c$

If you don’t know basic integration and differentiation, you then better catch up.

Extra resources:

Understanding Integration by part : Integration By Parts
Understanding Reduction formula: Reduction Formulas For Integration

Week 6
Maximum and Minimum, this will be quick…

Maximum and Minimum are methods used in determining the point of vertex of a parabola all these English means we are determining the apex (the highest point) of a curve in a graph and if you translate it to mathematical function, you're looking for the apex of a curve in a quadratic function. Clap, clap, clap, lol.

Bad news, we will be talking about this topic in the context of calculus. But the principles are closely the same, but if you wanted to see how this works in quadratic equation, it is very simple though, you can watch the video in the extra resources for week.

For calculus **Extremum values** refer to the highest or lowest values (maximum or minimum) that a function can reach, either within an interval (from point a to b, i.e [a, b]) or on the entire domain.

These points are crucial in fields like economics, engineering, and science, as they allow us to **optimize** outcomes, such as minimizing costs or maximizing efficiency

Types of Extrema

- **Global (Absolute) Maximum and Minimum**: The highest or lowest values a function takes over its entire domain.
- **Local (Relative) Maximum and Minimum**: Points where the function reaches a high or low value within a small neighborhood around that point, but not necessarily the highest or lowest value overall.

How do you find Extrema (Maxima/Maximum or Minima/Minimum)

In quadratic equation $f(x) = ax^2 + bx + c = 0$ and if $a > 0$ therefore the parabola (curve on graph) would open up like “U” and this would have a “Minimum point” and if $a < 0$ then the parabola would open down like “n” and would have “Maximum point” the highest point(apex) of the curve

I said this would be quick, apologies, we will need more lines…

In calculus context it is slightly different, when you are given a function f(x) you can find local Maximum0/Minimum using the following methods.

- Find the first derivative for f(x) → f’(x)
  - $f(x) = 3x^3 - 4x^2 + 8$, the first derivative would be $f'(x) = 9x^2 - 8x$
- Next, set f’(x) = 0 to find C (the critical point) this is the point where the curve takes a turn or curves, if you can’t find this when the function is = 0, then there is no Min/Max because the line of the graphs would contd growing without turning at some point which should have been the critical point, let’s continue with our example:
  - Set $9x^2 - 8x = 0$, factor it out, we would have $x(9x - 8) = 0$, therefor x = 0 and x = 8/9 are our critical points
- Let's find the maxima and minima i.e. how the graph opens, up or down
  - To get this, find the second derivative of $f(x) = 3x^3 - 4x^2 + 8$
  - The first was $f'(x) = 9x^2 - 8x$
  - The second will be f’’(x) = 18x - 8
  - Next solve for the function at the critical points (c) that we found, when x = 0 and x = 8/9
  - f’’(0) = 18(0) - 8 = -8 and f’’(8/9) = 18(8/9) - 8 = 8
  - Therefore, this means that when the critical point was 0, x = -8 and - 8 < 0, this suggests that the function opens down “n” and has a local maxima, when the critical point is 8/9, x = 8, this function at this point opens up “U” and has a local minma.

Note: If f″(x)>0, the graph is **concave up** (shaped like a "U") around that point, suggesting a local minimum. If f″<0, the graph is **concave down** (shaped like an "n") around that point, suggesting a local maximum.

- Lastly finding the exact values at those critical points, not the concavity (opening up or down), just substitute the critical point values into the original function $3x^3 - 4x^2 + 8$
    - When x = 0, $f(0) = 3(0)^3 - 4(0)^2 + 8 = 8$ and when x = 8/9, $f(8/9) = 3(8/9)^3 - 4(8/9)^2 + 8 = 7.65$, approx 8

These Extremum (Maximum & Minimum) principles are the same for the following functions in calculus, sinusoidal, polynomial, rationals, and exponential,

There is still the global maxima and minima which deals with intervals, i would have like to write it… but, I will add them as extra resources

Week 7
Partial differentiation is a type of differentiation involving two variables, x and y, we can differentiate x in isolation of y, or vice versa. $\partial f/\partial x$ which is differentiate x in isolation of y and $\partial f/\partial y$, means differentiate y in the isolation of x

Example: $\sin(x^2+y)$, find $\partial f/\partial x$
Solution: This means we only find the derivatives of x and y will remain constant

- Using the chain rule for $\sin(x^2+y) = \cos(x^2+y) * 2x \Rightarrow 2x\cos(x^2+y)$
- Answer: $2x\cos(x^2+y)$

Note all differentiation rules and methods applies here, chain rule, quotient rule, product rule etc.

Types of Second Derivatives

- $\partial^2u/\partial x^2$ Differentiate with x twice.
- $\partial^2u/\partial y^2$ Differentiate with y twice.
- $\partial^2u/\partial x\partial y$ Differentiate first with x, then with y

- $\partial^2 u/\partial y \partial x$ Differentiate first with y, then with x.

Extra resources:

Partial Derivative | Multivariable Chain Rule

How to find Partial Derivative of a function

Week 8

In calculus, **increment** refers to the small change or increase in the value of a variable. It's often represented with the Greek letter Δ (delta). The concept of increments is fundamental in understanding derivatives and the behavior of functions.

Key Ideas:

1. **Increment in a Variable:** If x is a variable, an **increment in x** is the small change in its value, denoted as:

   $\Delta x = x_{new} - x_{old}$

   It represents the "step" or "change" in x.

2. **Increment in a Function:** If f(x) is a function, the corresponding change in the function's value due to the change in x is called the **increment in the function** and is written as:

   $\Delta f = f(x+\Delta x) - f(x)$

Cobb-Douglas Production Function

The **Cobb-Douglas production function** is a common economic model that describes the relationship between inputs (like labor and capital) and output (goods or services produced). It takes the form:

$Y = AL^{\alpha}K^{\beta}$

Where:

- Y is the output (production),
- L is the amount of labor used,
- K is the amount of capital used,
- A is a constant that represents technological level,
- α and β are constants representing the output elasticities of labor and capital.

a. Using Partial Derivatives in Cobb-Douglas:

To understand how the output changes with respect to labor or capital, we take the **partial derivatives** of the Cobb-Douglas function.

Partial derivative with respect to labor (L):

$\partial Y / \partial L = A\alpha L^{\alpha-1} K^{\beta}$

This derivative tells us how output changes if we increase labor while keeping capital constant.

Partial derivative with respect to capital (K)

$\partial Y / \partial K = A\boldsymbol{\alpha} L^{\alpha} K^{\beta - 1}$

1. This tells us how output changes if we increase capital while keeping labor constant.

b. Economic Interpretation:

- The **marginal product of labor** (MPL) is given by $\partial Y/\partial L$. It represents how much additional output can be produced by adding one more unit of labor.
- The **marginal product of capital** (MPK) is given by $\partial Y/\partial K$. It represents how much additional output can be produced by adding one more unit of capital.

This model is particularly useful because it helps economists analyze the effects of changing input quantities (labor and capital) on output, and is often used in production theory to explain economic growth.

Talking about **Tangents, Linear Approximation, and Differentials**:

- Includes:

Tangents: Finding the equation of the tangent plane to a surface.

In one of our note, i did explain that a target line is a straight line that touches a curve at a single point, and goes in the same direction as the curve at that point. The slope of a tangent line is the same as the slope of the curve at the point of tangency.

In order to find this tangent line from just seeing a curve we make use of x & y on the graphs and it's values.

If you have a function z= f(x, y), the tangent plane at a point (x_0,y_0) approximates the surface near that point.

For z = f(x,y), the tangent plane at (x_0,y_0,z_0) is:

$z-z_0=fx(x_0,y_0)(x-x_0) + fy(x_0,y_0)(y-y_0)$

Where:

- $fx(x_0,y_0)$: Partial derivative of f(x,y) with respect to x at (x_0,y_0)
- $fy(x_0,y_0)$: Partial derivative of f(x,y) with respect to y at (x_0,y_0)
- $Z_0 =f(x_0,y_0)$: The z-value of the surface at (x_0,y_0)

I would have love to solve one question, but this letter is crazily, long, would search for a video and add it

Linear approximation uses the tangent plane to estimate the value of a multivariable function near a known point. By replacing a complex surface with its simpler tangent plane, we can quickly calculate approximate values for the function when the variables change slightly. This method is especially handy for predictions and estimations when exact calculations are too complex.

Formula is:

$z=f(x_0,y_0) + fx(x_0,y_0)(x-x_0) + fy(x_0,y_0)(y-y_0)$

Differentials: Differentials represent small changes in a function's value resulting from small changes in its variables. They rely on partial derivatives to determine how much each variable contributes to the overall change. Differentials are a practical tool for estimating how a system behaves when inputs change slightly, making them valuable in error analysis and real-world applications.

$dz= f_x dx+ f_y dy$

Week 9

My concern with week 9 is that the multivariate TSE, are quite big, that putting them as note here would look bulky, but i will try and go on to put extra videos you can watch,

What is the Multivariate Taylor Series Expansion?

The **Taylor series** is a way to approximate a function near a specific point by expressing it as an infinite sum of terms. In the multivariable case, we expand functions with more than one variable (e.g., f(x,y)) around a point.

Why Use It?

- To approximate functions of several variables near a given point.

- Useful for analyzing behavior in complex systems (e.g., engineering, physics).
- Helps in numerical methods and optimization.

3. Key Components:

For a function $f(x, y)$, expanded around a point (x_0, y_0), the multivariate Taylor series includes:

1. **Zeroth-Order Term**:
   - $f(x_0, y_0)$: The value of the function at the expansion point.
2. **First-Order Terms**:
   - Linear terms based on partial derivatives:
   $$f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

   These describe the slope in the x- and y-directions.
3. **Second-Order Terms**:
   - Quadratic terms involving second partial derivatives:
   $$\frac{1}{2} f_{xx}(x_0, y_0)(x - x_0)^2 + f_{xy}(x_0, y_0)(x - x_0)(y - y_0) + \frac{1}{2} f_{yy}(x_0, y_0)(y - y_0)^2$$
4. **Higher-Order Terms**:
   - Cubic or higher-order terms involving third or higher derivatives. These are often ignored for simplicity in practical approximations.

The **Jacobian** is a determinant that plays a crucial role in multivariable calculus. It measures how functions transform as their variables change and is especially useful in:

- **Coordinate transformations** (e.g., from Cartesian to polar or spherical coordinates).
- **Finding inverse functions**.
- **Solving systems of equations**.

For indepth under for the following:

Jacobiam, Minimum & Maximum under partial derivatives, and Lagrange multiplier, i will drop video on them.

Extra resources:

- Jacobian| jacobian transformation|differential calculus
- Local Extrema, Critical Points, & Saddle Points of Multivariable Function…
- 🟡15a - Lagrange's Multipliers: One Constraints - Find the maximum an…