FIBONACCI NUMBERS

The Fibonacci numbers are nature's numbering system. They appear many places in nature, from the leaf arrangement in plants, to the pattern of the florets of a flower, the bracts of a pinecone, or the scales of a pineapple. The Fibonacci numbers are therefore applicable to the growth of many living things, including a single cell, a grain of wheat, a hive of bees, and even mankind.

Fibonacci was known in his time and is still recognized today as the "greatest European mathematician of the middle ages." He was born in the 1170's and died in the 1240's and there is now a statue commemorating him located at the Leaning Tower end of the cemetery next to the Cathedral in Pisa.
His full name was Leonardo of Pisa, or Leonardo Pisano in Italian since he was born in Pisa. He called himself Fibonacci which was short for Filius Bonacci, standing for "son of Bonacci", which was his father's name. Leonardo's father (Guglielmo Bonacci) was a kind of customs officer in the North African town of Bugia, now called Bougie (Bey-jai-ya). So, Fibonacci grew up with a North African education under the Moors and later travelled extensively around the Mediterranean coast.

He then met with many merchants and learned of their systems of doing arithmetic. He soon realized the many advantages of the "Hindu-Arabic" system over all the others. He was one of the first people to introduce the Hindu-Arabic number system into Europe-the system we now use today- based of ten digits with its decimal point and a symbol for zero: 1 2 3 4 5 6 7 8 9. and 0 rather than the limiting Roman numerical system that was used at the time. His book on how to do arithmetic in the decimal system, called Liber abbaci (meaning Book of the Abacus or Book of calculating) completed in 1202 persuaded many of the European mathematicians of his day to use his "new" system. The book goes into detail (in Latin) with the rules we all now learn in elementary school for adding, subtracting, multiplying and dividing numbers altogether with many problems to illustrate the methods in detail.

Fibonacci posed the following question:

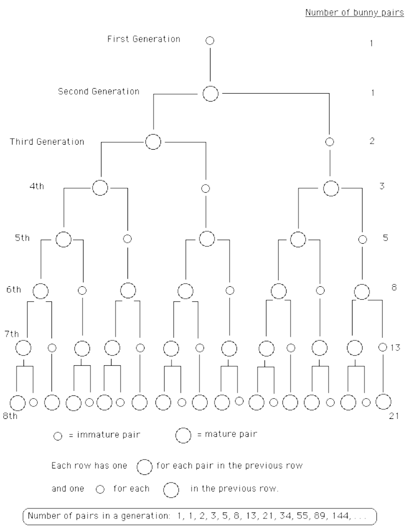
If a pair of rabbits is placed in an enclosed area, how many rabbits will be born there if we assume that every month a pair of rabbits produces another pair, and that rabbits begin to bear young two months after their birth?

This apparently innocent little question has as an answer a certain sequence of numbers, known now as the Fibonacci sequence, which has turned out to be one of the most interesting ever written down. It has been rediscovered in an astonishing variety of forms, in branches of mathematics way beyond simple arithmetic. Its method of development has led to far-reaching applications in mathematics and computer science.

Now in the Fibonacci rabbit situation, there is a lag factor; each pair requires some time to mature. So we are assuming

- maturation time = 1 month
- gestation time = 1 month

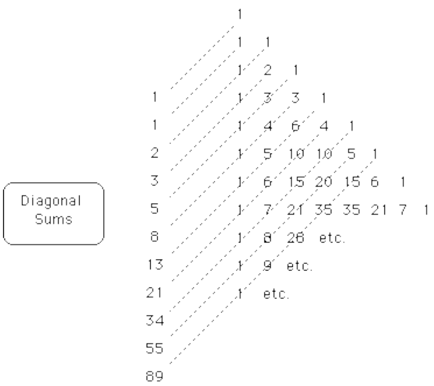
If you were to try this in your backyard away from other factors like predators and other survival issues, here's what would happen:



Each successive number in this sequence is obtained by adding the two previous numbers together. And, deleting complicating details like the fact that rabbits eventually grow old and die, this sequence does an admirable job at modelling how populations grow. But the numbers in Fibonacci’s sequence have a life far beyond rabbits and show up in the most unexpected places.

Now let's look at another reasonably natural situation where the same sequence "mysteriously" pops up. Go back to 17th century France. Blaise Pascal is a young Frenchman, scholar who is torn between his enjoyment of geometry and mathematics and his love for religion and theology. In one of his worldlier moments he is consulted by a friend, a professional gambler, the Chevalier de Mé ré , Antoine Gombaud. The Chevalier asks Pascal some questions about plays at dice and cards, and about the proper division of the stakes in an unfinished game. Pascal's response is to invent an entirely new branch of mathematics, the theory of probability. This theory has grown over the years into a vital 20th century tool for science and social science although it had been described centuries earlier by Chinese mathematician Yanghui (about 500 years earlier, in fact) and the Persian astronomer-poet Omar Khayyám.

Pascal's work leans heavily on a collection of numbers now called Pascal's Triangle, and represented like this:



and we get 1, 1, 2, 3, 5, 8, 13, . . . the Fibonacci sequence!

Fibonacci could not have known about this connection between his rabbits and probability theory - the theory didn't exist until 400 years later.

It is quite amazing that the Fibonacci sequences appear so often in biological settings, such as branching in trees, arrangement of leaves on a stem, the fruitlets of a pineapple, the flowering of artichoke, an uncurling fern and the petals in lilies and buttercups. In addition, numerous claims of Fibonacci numbers or golden sections in nature are found in popular sources, e.g. relating to weather patterns like hurricanes, the spirals of shells, and the curve of waves etc. The Fibonacci numbers are even found in the family tree of honeybees!

Plants do not know about this sequence - they just grow in the most efficient ways. Many plants show the Fibonacci numbers in the arrangement of the leaves around the stem. Some pine cones and fir cones also show the numbers, as do daisies and sunflowers. Sunflowers can contain seeds that number 89, or even 144. Many other plants, such as succulents, also show the numbers. Some coniferous trees show these numbers in the bumps on their trunks. And palm trees show the numbers in the rings on their trunks.

Why do these arrangements occur? In the case of leaf arrangement, or phyllotaxis, some of the cases may be related to maximizing the space for each leaf, or the average amount of light falling on each one. Even a tiny advantage would come to dominate, over many generations. In the case of close-packed leaves in cabbages or growth in Romanesque broccoli/cauliflower and succulents the correct arrangement may be crucial for availability of space.

So, nature isn't trying to use the Fibonacci numbers: they are appearing as a by-product of a deeper physical process. That is why the spirals are imperfect.
The plant is responding to a natural physical, energetic template, not to a mathematical rule.

Connection Between the Golden Ratio and the Fibonacci Sequence

Okay, but what about the Golden Ratio? How does that figure into this? I know it might seem totally unrelated but check this out. Let's create a new sequence of numbers by dividing each number in the Fibonacci sequence by the previous number in the sequence. Remember, the sequence is

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...

So, dividing each number by the previous number gives: 1 / 1 = 1, 2 / 1 = 2, 3 / 2 = 1.5, and so on up to 144 / 89 = 1.6179. The resulting sequence is:

1, 2, 1.5, 1.666..., 1.6, 1.625, 1.615…, 1.619…, 1.6176…, 1.6181…, 1.6179…

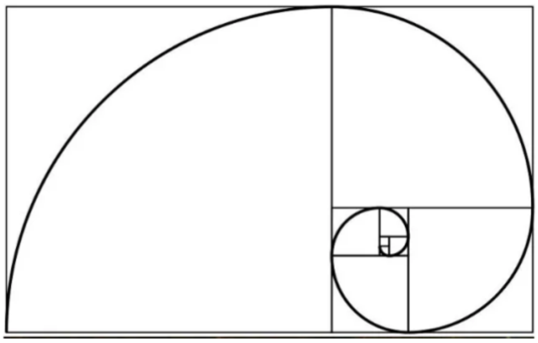
But do you notice anything about those numbers? Perhaps the fact that they keep oscillating around and getting tantalizingly closer and closer to 1.618?—the value of phi: the golden ratio! Indeed, completely unbeknownst to Fibonacci, his solution to the rabbit population growth problem has a deep underlying connection to the golden ratio that artists and architects have used for thousands of years!

Closely related to the Fibonacci Sequence, the Golden Ratio describes the perfectly symmetrical relationship between two proportions.

Approximately equal to a 1:1.61 ratio, the Golden Ratio can be illustrated using a Golden Rectangle (like our credit cards): a large rectangle consisting of a square (with sides equal in length to the shortest length of the rectangle) and a smaller rectangle. If you remove this square from the rectangle, you'll be left with another, smaller Golden Rectangle. This could continue infinitely, like Fibonacci numbers.

Knowledge of the golden section, ratio and rectangle goes back to the Greeks & Egyptians, who based their most famous work of art on them: the Parthenon is full of golden rectangles as are the Great Pyramids. The Greek followers of the mathematician and mystic Pythagoras even thought of the golden ratio as divine.
The unique properties of this shape, a rectangle in which the ratio of the sides a/b is equal to the golden mean (phi), can result in a nesting process that can be repeated into infinity and which takes on the form of a spiral. It's called the logarithmic spiral, and it abounds in nature.

Snail shells and nautilus shells follow the logarithmic spiral, as does the cochlea of the inner ear. It can also be seen in the horns of certain goats, and the shape of certain spider's webs.

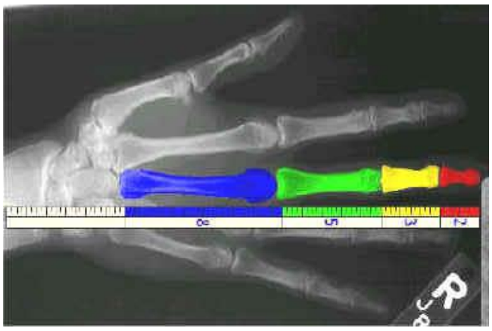


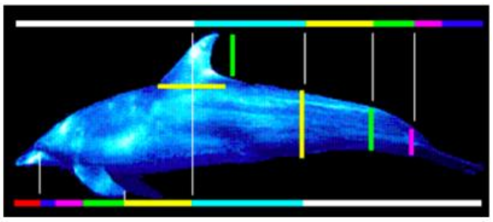
Not surprisingly, spiral galaxies also follow the familiar Fibonacci pattern. The Milky Way has several spiral arms, each of them a logarithmic spiral of about 12 degrees. As an interesting aside, spiral galaxies appear to defy Newtonian physics. As early as 1925, astronomers realized that, since the angular speed of rotation of the galactic disk varies with distance from the center, the radial arms should become curved as galaxies rotate. Subsequently, after a few rotations, spiral arms should start to wind around a galaxy. But they don't — hence the so-called winding problem. The stars on the outside, it would seem, move at a velocity higher than expected — a unique trait of the cosmos that helps preserve its spiral shape.

Faces, both human and nonhuman, abound with examples of the Golden Ratio. The mouth and nose are each positioned at golden sections of the distance between the eyes and the bottom of the chin. Similar proportions can be seen from the side, and even the eye and ear itself (which follows along a spiral).

It's worth noting that every person's body is different, but that averages across populations tend towards phi. It has also been said that the more closely our proportions adhere to phi, the more "attractive" those traits are perceived. As an example, the most "beautiful" smiles are those in which central incisors are 1.618 wider than the lateral incisors, which are 1.618 wider than canines, and so on. It's quite possible that, from an evo-psych perspective, that we are primed to like physical forms that adhere to the golden ratio — a potential indicator of reproductive fitness and health.

Looking at the length of our fingers, each section — from the tip of the base to the wrist — is larger than the preceding one by roughly the ratio of phi.





Even our bodies exhibit proportions that are consistent with Fibonacci numbers. For example, the measurement from the navel to the floor and the top of the head to the navel is the golden ratio. Animal bodies exhibit similar tendencies, including dolphins (the eye, fins and tail all fall at Golden Sections), starfish, sand dollars, sea urchins, ants, and honey bees.

Even the microscopic realm is not immune to Fibonacci. The DNA molecule measures 34 angstroms long by 21 angstroms wide for each full cycle of its double helix spiral. These numbers, 34 and 21, are numbers in the Fibonacci series, and their ratio 1.6190476 closely approximates Phi, 1.6180339.

As a result of the unique properties of this golden proportion, many view the ratio as sacred or divine and as a door to a deeper understanding of beauty and spirituality in life, unveiling a hidden harmony or connectedness in so much of what we see.