

Algebra test solution

1.

$-$: Subtraction
- : Negative
+ : Addition
 $\times$: Multiplication
 $\div$ or $/$: Division

Rule for multiplying or dividing integers:

a. Same signs, you get a plus

$$- \times - = + \quad + \times + = +$$

b. Different signs, you get a minus

$$- \times + = - \quad + \times - = -$$

$$-10 \times +5 = -50$$

2.

Rule for adding and subtracting integers

$-$: Subtraction
- : Negative

a. Same signs add and keep the number sign

b. If you have different signs, subtract and keep the same of the number that has more negative or more positive.

c. Subtracting a negative is the same thing as adding a positive:

$$\text{So, } - - = + +$$

d. Subtracting a positive is the same thing as adding a negative

$$- + = + - \text{ or } - = + - \text{ (Sometimes, the plus sign here } - + \text{ is omitted)}$$

$$-10 + 5.$$

Different signs, so subtract. $10 - 5 = 5$

Here, we have more negative, so keep the negative

$$-10 + 5 = -5$$

3.

$$\frac{-16}{-4} = +4$$

4.

Rules to use for this problem:

Subtracting a negative is the same thing as adding a positive:

So, $- - = + +$

$$-1 - -8 = -1 + +8$$

If you have different signs, subtract and keep the same of the number that has more negative or more positive

$$8 - 1 = 7$$

Since there are more positive in $-1 + +8$ we keep the positive.

$$-1 + +8 = +7$$

5.

First, do the two multiplications, and then add.

$$|-6 \times 2 + -4 \times -3| = |-12 + 12| = |0| = 0$$

6.

Rules for simplifying expressions:

When simplifying expressions, put all like terms together and add or subtract.

Like terms are terms with the same variable and same exponent.

Example: $3x$ and $7x$,

Never add unlike terms.

For example $11x$ and 11

a)

$$4x + 5 + 2x = 4x + 2x + 5 = 6x + 5$$

b)

$$\begin{aligned} -6x + 4z + -3x - 10z &= -6x + -3x + 4z - 10z && \text{Put like terms together} \\ &= -6x + -3x + 4z + -10z && \text{Change a minus + into plus negative} \\ &= -9x + -6z \end{aligned}$$

c)

$$\begin{aligned} 10y - 2y - 3z - 9z - 6y &= 10y + + 2y + -3z + - 9z + -6y && (- - = + + \text{ and } - = + -) \\ &= 10y + + 2y + -6y + -3z + -9z \\ &= 12y + -6y + -12z \\ &= 6y + -12z \end{aligned}$$

d)

Rule for multiplying numbers with exponents

$$a^n \times a^m = a^{m+n}$$

Just add the exponents and keep the same base.

Rule for dividing numbers with exponents

$$\frac{a^n}{a^m} = a^{n-m}$$

Just subtract the denominator from the numerator and keep the same base

$$\begin{aligned} x^5 \times y^4 \times x^{-7} \times y^{10} &= x^5 \times x^{-7} \times y^4 \times y^{10} \\ &= x^{5+-7} \times y^{4+10} \\ &= x^{-2} \times y^{14} \end{aligned}$$

e)

$$\frac{x^8}{x^2} = x^{8-2} = x^6$$

f)

$$(4x)^3 = 4x \times 4x \times 4x = 4 \times 4 \times 4 \times x \times x \times x = 64 x^3$$

e)

$$\begin{aligned} (-5c)^4 &= -5c \times -5c \times -5c \times -5c \\ &= -5 \times -5 \times -5 \times -5 \times c \times c \times c \times c \\ &= 25 \times 25 \times c^4 \\ &= 625 \times c^4 \end{aligned}$$

7.

Associative property:

$$(a + b) + c = a + (b + c)$$

$$(a \times b) \times c = a \times (b \times c)$$

Answer is C. $(5 \times 7) \times 2 = 5 \times (7 \times 2)$

B. $5 + 7 = 7 + 5$ is the commutative property.

A. $5 \times 7 + 2 = 2 + 5 \times 7$ is still a tricky version of the commutative property.

D. $2 + 7x$ is completely irrelevant.

8.

Answer is C.

Identity property of multiplication, distributive property, and commutative property of addition

The distributive property is shown in part c)

The identity property of multiplication is shown in part b)

The commutative property of addition is shown in part a)

Identity property of addition: $a + 0 = a$

Distributive property: $a (c + d) = ac + ad$

9.

a)

$$x - 2 = -7$$

$$x - 2 + 2 = -7 + 2$$

$$x + 0 = -5$$

$$x = -5$$

b)

$$x - -2 = -7$$

$$x + +2 = -7$$

$$x + +2 - 2 = -7 - 2$$

$$x + 0 = -7 + -2$$

$$x = -9$$

c)

$$(2/3)x = 6$$

$$\frac{2}{3}x = 6$$

$$\frac{3}{2} \times \frac{2}{3}x = 6 \times \frac{3}{2}$$

$$\frac{6}{6} \times x = \frac{6}{1} \times \frac{3}{2}$$

$$x = \frac{18}{2}$$

$$x = 9$$

d)

$$-2x + 5 = -11$$

$$-2x + 5 - 5 = -11 - 5$$

$$-2x = -11 + -5$$

$$-2x = -16$$

$$x = 8$$

Divide both sides by -2

e)

$$ax + b = -e$$

$$ax + b - b = -e - b$$

$$ax = -e + -b$$

$$\frac{ax}{a} = \frac{-e + -b}{a}$$

$$x = \frac{-e + -b}{a}$$

10.

a)

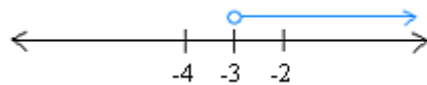
$$3x - 2 > -11$$

$$3x - 2 + 2 > -11 + 2$$

$$3x > -9$$

$$\frac{3x}{3} > \frac{-9}{3}$$

$$x > -3$$



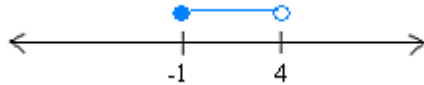
b) $x \neq 5$

if $x \neq 5$, $x > 5$ or $x < 5$. The graph is shown below:



c) $-1 \leq x < 4$

This means x is between -1 and 4 . -1 is included in the solution, but 4 is not.
Notice that the circle on the left is shaded to show that -1 is included in the solution.

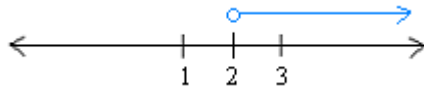


d) $-5f < -10$

$$\frac{-5}{-5} f > \frac{-10}{-5}$$

$$f > 2$$

Notice that the direction of the inequality was switched. That is what you do when you divide or multiply by a negative number



11.

A number is in scientific notation if it is written as

$$a \times 10^n \quad \text{a is at least 1 and less than 10}$$

Here is the technique:

Move the decimal point until you get a number that is at least 1 and less than 10.

The number of places you moved the point is your exponent or n .

If the number is at least 1, the exponent is at least zero ($1 = 1 \times 10^0$)

Usually, you move the point to the left.

If the number is smaller than 1, the exponent is a negative number. Usually, you move the point to the right.

When you don't see the point, put it at the end.

a) 0.000000000245

Move the decimal point 11 places to the right to put it between 2 and 4.
The resulting number is 2.45 and $n = -11$ since the number is smaller than 1.

$$0.0000000000245 = 2.45 \times 10^{-11}$$

b) 890000000

$$890000000.$$

Move the decimal point 8 places to the left to put it between 8 and 9.
The resulting number is 8.9 and $n = 8$ since the number is at least 1.

$$890000000. = 8.9 \times 10^8$$

12.

a)

$$\begin{aligned} & 9z^8 + 2z^5 + 4z^2 + 2 \text{ and } 5z^8 + -6z^5 + -4z^2 + -1z + -8 \\ &= 9z^8 + 2z^5 + 4z^2 + 2 + 5z^8 + -6z^5 + -4z^2 + -1z + -8 \\ &= 9z^8 + 5z^8 + 2z^5 + -6z^5 + 4z^2 + -4z^2 + -1z + 2 + -8 \\ &= (9 + 5)z^8 + (2 + -6)z^5 + (4 + -4)z^2 + -1z + 2 + -8 \\ &= 14z^8 + -4z^5 + (0)z^2 + -1z + -6 \\ &= 14z^8 + -4z^5 + -1z + -6 \end{aligned}$$

b)

$$\begin{aligned} & z^6 + 5z^4 + -10z^3 + -7 \text{ and } -z^6 + 8z^4 + 4z^3 - 8 \\ &= z^6 + -z^6 + 5z^4 + 8z^4 + -10z^3 + 4z^3 + -7 + -8 \\ &= 13z^4 + -6z^3 + -15 \end{aligned}$$

13. a) $4x^3(2x^2 + x - 5)$

$$4x^3 \times 2x^2 + 4x^3 \times x + 4x^3 \times -5 = 8x^5 + 4x^4 + -20x^3$$

b) $(4x^3 + 4x)(2x^2 + x - 5)$

$$(4x^3 + 4x)(2x^2 + x - 5)$$

$$= 4x^3 \times 2x^2 + 4x^3 \times x + 4x^3 \times -5 + 4x \times 2x^2 + 4x \times x + 4x \times -5$$

$$= 8x^5 + 4x^4 + -20x^3 + 8x^3 + 4x^2 + -20x$$

$$= 8x^5 + 4x^4 + -12x^3 + 4x^2 + -20x$$

14. $x^3 - 6x^2 + 5x - 30 / x - 6$

$$\begin{array}{r} x^2 \\ x - 6 \overline{) x^3 - 6x^2 + 5x - 30} \end{array}$$

Ask yourself. How many times x goes into x^3 ? It goes x^2 times. Put x^2 where you see it. This is shown in blue

$$\begin{array}{r} x^2 \\ x - 6 \overline{) x^3 - 6x^2 + 5x - 30} \end{array}$$

Multiply x^2 by $x-6$ and subtract from $x^3 - 6x^2 + 5x - 30$.
 $x^2 \times (x-6) = x^3 + -6x^2$

$$\begin{array}{r} x^2 \\ x - 6 \overline{) x^3 - 6x^2 + 5x - 30} \\ - (x^3 + -6x^2) \end{array}$$

Multiply x^2 by $x-6$ and subtract from $x^3 - 6x^2 + 5x - 30$.

$$\begin{array}{r}
 x^2 \\
 x - 6 \overline{) x^3 - 6x^2 + 5x - 30} \\
 \underline{- x^3 + 6x^2} \\
 0 + 0 + 5x - 30
 \end{array}$$

$$- (x^3 + -6x^2) = -x^3 + 6x^2$$

Add the two lines.

$$\begin{array}{r}
 x^2 + 5 \\
 x - 6 \overline{) x^3 - 6x^2 + 5x - 30} \\
 \underline{- x^3 + 6x^2} \\
 5x - 30
 \end{array}$$

Ask yourself. How many times x goes into 5x ? It goes 5 times. Put 5 where you see it. This is shown in red

$$\begin{array}{r}
 x^2 + 5 \\
 x - 6 \overline{) x^3 - 6x^2 + 5x - 30} \\
 \underline{- x^3 + 6x^2} \\
 5x - 30
 \end{array}$$

Multiply 5 by x-6 and subtract the answer from 5x + -30
 $5 \times (x - 6) = 5x + -30$

$$\begin{array}{r}
 x^2 + 5 \\
 x - 6 \overline{) x^3 - 6x^2 + 5x - 30} \\
 \underline{- x^3 + 6x^2} \\
 5x - 30 \\
 \underline{- 5x + 30} \\
 0 + 0
 \end{array}$$

$$- (5x + -30) = -5x + 30$$

Add the two lines

15.

GCF (15, 30)

List the factors of 15: **1, 3, 5, 15**

List the factors of 30: **1 2 3 5 6 10 15 30**

Common factors are shown in bold. They are 1, 3, 5, and 15.

However, the greatest common factor is 15

$$\text{GCF}(15x^3, 30x^2)$$

$$\text{GCF}(15, 30) = 15$$

Just get $\text{GCF}(x^3, x^2)$

List the factors of x^3 : 1, x, x^2 , and x^3

List the factors of x^2 : 1, x, and x^2

The common factors are 1, x, and x^2 but the greatest common factor is x^2

$$\text{GCF}(15x^3, 30x^2) = 15x^2$$

16.

$$\text{a) } 12x^2 + 24x^3 + -18x$$

Find the greatest common factor (GCF)

The biggest number that can go into 12, 24, and 18 is 6, so $\text{GCF}(12, 24, 18) = 6$

The biggest number that can go into x^2, x^3, x is x so $\text{GCF}(x^2, x^3, x) = x$

Therefore, the $\text{GCF}(12x^2 + 24x^3 + -18x) = 6x$

Factor this expression.

$$\begin{aligned} 12x^2 + 24x^3 + -18x &= 6x(2x + 4x^2 - 3) \\ &= 6x(4x^2 + 2x - 3) \end{aligned}$$

Factor $4x^2 + 2x - 3$

This problem is best solved with the quadratic formula since the solutions are not

Integers and then say that the factorization form is $a(x - x_1)(x - x_2)$

This problem turns out to be very complicated.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The standard form of a quadratic equation is $ax^2 + bx + c$

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \times 4 \times -3}}{2 \times 4}$$

$$x = \frac{-2 \pm \sqrt{4 + -4 \times 4 \times -3}}{2 \times 4}$$

$$x = \frac{-2 \pm \sqrt{4 + -4 \times 4 \times -3}}{8}$$

$$x = \frac{-2 \pm \sqrt{4 + -16 \times -3}}{8}$$

$$x = \frac{-2 \pm \sqrt{4 + 48}}{8}$$

$$x = \frac{-2 \pm \sqrt{52}}{8}$$

$$x = \frac{-2 \pm 2\sqrt{13}}{8}$$

$$x = \frac{-2 \pm 2\sqrt{13}}{2 \times 4}$$

$$x = \frac{2 \times (-1 \pm \sqrt{13})}{2 \times 4}$$

$$x = \frac{-1 \pm \sqrt{13}}{4}$$

$$x_1 = \frac{-1 + \sqrt{13}}{4}$$

$$x_2 = \frac{-1 - \sqrt{13}}{4}$$

The factorization form is $a(x - x_1)(x - x_2) =$

$$4 \left(x - \frac{-1 + \sqrt{13}}{4} \right) \left(x - \frac{-1 - \sqrt{13}}{4} \right) =$$

Putting it together, we get:

$$\begin{aligned} 12x^2 + 24x^3 + -18x &= 6x \times 4 \left(x - \frac{-1 + \sqrt{13}}{4} \right) \left(x - \frac{-1 - \sqrt{13}}{4} \right) \\ &= 24x \left(x - \frac{-1 + \sqrt{13}}{4} \right) \left(x - \frac{-1 - \sqrt{13}}{4} \right) \end{aligned}$$

b) $x^2 - 3x - 18$

Just look for factors of -18 that adds up to -3

$$-18 = 6 \times -3$$

$$\mathbf{-18 = -6 \times 3 \quad \text{This one adds up to -3 (-6 + 3 = -3)}}$$

$$-18 = 9 \times -2$$

$$-18 = -9 \times 2$$

$$-18 = -18 \times -1$$

$$-18 = 18 \times -1$$

$$\text{So, } x^2 - 3x - 18 = (x + -6) \times (x + 3)$$

c)

First get the GCF

$$x^6 + 13x^5 + 42x^4$$

The biggest number that can go into 1, 13, and 42 is 1, so $\text{GCF}(1, 13, 42) = 1$

The biggest number that can go into x^6, x^5, x^4 is x^4 so $\text{GCF}(x^6, x^5, x^4) = x^4$

Therefore, the $\text{GCF}(x^6 + 13x^5 + 42x^4) = x^4$

Factor this expression.

$$x^6 + 13x^5 + 42x^4 = x^4 (x^2 + 13x + 42)$$

Now factor $x^2 + 13x + 42$

Just look for factors of -18 that adds up to -3

$$42 = 2 \times 21$$

$$42 = -2 \times -21$$

$$42 = 7 \times 6 \quad \mathbf{\text{This one adds up to 13 (7 + 6 = 13)}}$$

$$42 = -7 \times -6$$

$$\text{So, } x^2 + 13x + 42 = (x + 6) \times (x + 7)$$

Putting it together, we get:

$$x^6 + 13x^5 + 42x^4 = x^4 (x^2 + 13x + 42) = x^4 (x + 6) \times (x + 7)$$

17.

To solve for x, make $(x + -6) \times (x + 3)$ equals 0

$$(x + -6) \times (x + 3) = 0$$

$$x + -6 = 0$$

$$x + -6 + 6 = 0 + 6$$

$$x = 6$$

$$x + 3 = 0$$

$$x + 3 - 3 = 0 - 3$$

$$x = -3$$

To solve for x, make $x^4 (x + 6) \times (x + 7) = 0$

$$x^4 = 0, \text{ so } x = 0$$

$$x + 6 = 0$$

$$x + 6 - 6 = 0 - 6$$

$$x = -6$$

$$x + 7 = 0$$

$$x + 6 - 7 = 0 - 7$$

$$x = -7$$

18.

Key concepts: counting principle

If task #1 can be done in n ways and task #2 can be done in m ways.

Task #1 and task #2 can be done in n times m ways.

4 people will sit on 4 chairs.

Task #1 consists of sitting the first person.

The first person has 4 choices.

After the first person sits, there are 3 chairs left, so the second person has 3 choices
 The third person has 2 choices
 The last person has 1 choice.

The number of ways 4 people can sit is $4 \times 3 \times 2 \times 1 = 24$.

19. Box and Whiskers plot with 12, 5, 2, 20, 7, 24, 10, 21

First, put the data in order from least to greatest.

2, 5, 7, 10, 12, 20, 21, 24

Lower extreme is 2.

Upper extreme is 24

Look for the median of this set and call it Median #1. The median is the number in the Middle.

2, 5, 7, 10, Median #1 12, 20, 21, 24

What number is between 10 and 12? It is 11, so the median is 11.

Then, look for the median of the set on the left of median #1. This set is 2, 5, 7, 10

2, 5, Median 7, 10

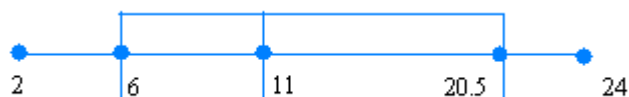
What number is between 5 and 7? It is 6, so the median is 6. This number is called the lower quartile.

Finally, look for the median of the set on the right of this median #1. This set is 12, 20, 21, 24

12, 20, Median 21, 24

What number is between 20 and 21? It is 20.5, so the median is 20.5. This number is called the upper quartile.

The graph is shown below:



20.

$$2x + 5 > 12$$

$$2x + 5 - 5 > 12 - 5$$

$$2x > 7$$

$$\frac{2}{2}x > \frac{7}{2}$$

$$x > \frac{7}{2}$$

$$x > 3.5$$



a) There are 3 white balls and 10 balls in the whole bag

So the probability is $\frac{3}{10}$ or $\frac{30}{100}$ or 30%

b) Blue or red equals to 7 balls

So the probability is $\frac{7}{10}$ or $\frac{70}{100}$ or 70%

c) Two red balls. This question is harder than it looks

This means that on the first draw, you pick a red ball and put it in your pocket. Then on the second draw, you pick a red ball again and put it in your pocket.

The probability of picking a red ball the first time is $\frac{5}{10}$

Since you put the ball in your pocket, there are now 9 balls, but 4 red.

The probability of picking a red ball again is $\frac{4}{9}$

Finally, you multiply the probability

$$\frac{5}{10} \times \frac{4}{9} = \frac{20}{90} = \frac{2}{9} = 0.2222 = 22.22\%$$

d) A blue ball and a white ball

This means that on the first draw, you pick a blue ball and put it in your pocket. Then on the second draw, you pick a white ball and put it in your pocket

The probability of picking a blue ball the first time is $\frac{2}{10}$

Since you put the ball in your pocket, there are now 9 balls.

The probability of picking a white ball is $\frac{3}{9}$

Finally, you multiply the probability

$$\frac{2}{10} \times \frac{3}{9} = \frac{6}{90} = 0.0666 = 6.66\%$$

21.

$$x + y = 9$$

$$x - y = 7$$

Let's solve this by elimination. We can eliminate y by adding the left sides and the right sides

$$x + x + y - y = 9 + 7$$

$$2x + 0 = 16$$

$$2x = 16$$

$$\text{So } x = 8.$$

Replace x into either equation. Let us replace into the first one.
We get:

$$8 + y = 9$$

$$8 - 8 + y = 9 - 8$$

$$0 + y = 1$$

$$\text{So, } y = 1$$

Therefore, the solution is (8, 1)

And indeed after replacing this into the system, we get

$$x + y = 9$$

$$x - y = 7$$

$$8 + 1 = 9$$

$$8 - 1 = 7$$

Here is how to graph. The graph is shown at the end of the explanation.

Write both equations in standard form. $y = mx + b$

$$x + y = 9$$

$$x - x + y = 9 - x$$

$$y = 9 - x$$

Compare with $y = mx + b$.

b is the y -intercept, so it goes on the y -axis.

Here $b = 9$

m is the slope. Here it is $-1 = -\frac{1}{1}$

First put 9 on the y -axis. This is shown with a red dot.

Then, use $-\frac{1}{1}$ to locate the second point.

To locate the second point, you always start at the first point you put on the Graph.

If the slope is positive, you go up. Otherwise, if it is negative you go down.

The number on top tells you how much you go up and the number at the bottom tells you how much you go from left to right.

$-\frac{1}{1}$ means to go down 1 unit and go to the right 1 unit

You end up where you see the black dot.

Now graph, $x - y = 7$

gain, $x - y = 7$ is equivalent to

$$x - y + y = 7 + y$$

$$x = 7 + y$$

$$x - 7 = 7 - 7 + y$$

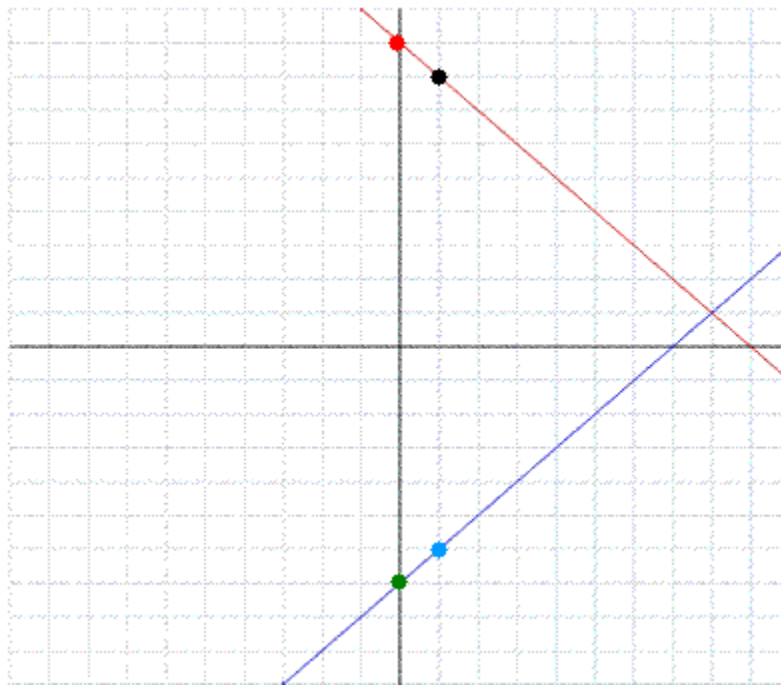
$$x - 7 = y$$

This time $b = -7$. it goes where you see the green dot.

$$m = 1 \text{ or } \frac{1}{1}$$

$\frac{1}{1}$ means to go up 1 unit and to the right 1 unit. You end up where you see

the blue dot



Check for yourself that the graphs meet at (8,1) and that is the solution.

$$4x - 3y = 10$$

$$8x + 5y = -10$$

Let's solve this by elimination. We can eliminate x by multiplying the first equation by -2.

We get:

$$-8x + 6y = -20$$

$$8x + 5y = -10$$

Add the left sides and the right sides.

$$\begin{array}{r} -8x + 8x + 6y + 5y = -20 + -10 \\ 11y = -30 \end{array}$$

$$y = -\frac{30}{11} = -2.7272$$

Replace y into the second to get x

$$8x + 5 \times -2.7272 = -10$$

$$8x + -13.63 = -10$$

$$8x + -13.63 + 13.63 = -10 + 13.63$$

$$8x = 3.63$$

$$x = \frac{3.63}{8}$$

$$x = 0.45$$

Now, let's graph the system

$$4x - 3y = 10$$

$$8x + 5y = -10$$

First, write $4x - 3y = 10$ as $y = mx + b$

$$4x - 4x - 3y = 10 - 4x$$

$$-3y = 10 - 4x$$

$$y = -\frac{10}{3} + \frac{-4x}{-3}$$

$$y = -\frac{10}{3} + \frac{4x}{3}$$

$b = -\frac{10}{3} = -3.33$, so put it where you see the red dot.

The slope is $\frac{4}{3}$, so go up 4 units and then go to the right 3 units.

You end up where you see the black dot.

Now write $8x + 5y = -10$ as $y = mx + b$

$$8x + 5y = -10$$

$$8x - 8x + 5y = -10 - 8x$$

$$5y = -10 - 8x$$

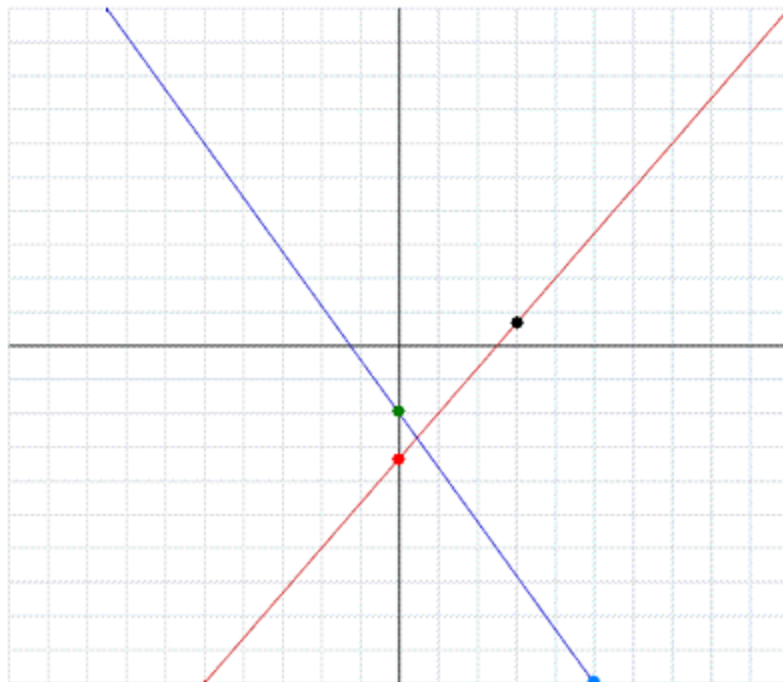
$$y = -\frac{10}{5} + \frac{-8x}{5}$$

$b = -\frac{10}{5} = -2$ so put it where you see the green dot.

The slope is $-\frac{8}{5}$, so go down 8 units and then go to the right 5 units.

You end up where you see the blue dot.

The graph is show below:



Check for yourself that the graphs meet at (0.45, -2.7272) and that is the solution

22. a) $(25x^6 36 z^7) / (50x^8 6z^5)$

$$\frac{25 x^6 \times 36 z^7}{50 x^8 \times 6 z^5} = \frac{25 x^6 \times 6 \times 6 \times z^5 \times z^2}{2 \times 25 \times x^6 \times x^2 \times 6 \times z^5}$$

$$= \frac{\cancel{25} x^{\cancel{6}} \times \cancel{6} \times 6 \times \cancel{z^5} \times z^2}{2 \times \cancel{25} \times \cancel{x^6} \times x^2 \times \cancel{6} \times \cancel{z^5}}$$

$$= \frac{6 \times z^2}{2 \times x^2}$$

$$= \frac{3 \times 2 \times z^2}{2 \times x^2}$$

$$= \frac{3 \times \cancel{2} \times z^2}{\cancel{2} \times x^2}$$

$$= \frac{3 \times z^2}{x^2}$$

$$\text{b) } x^6 + 13x^5 + 42x^4 / x^3 (x + 6)$$

$$x^6 + 13x^5 + 42x^4 = x^4 (x^2 + 13x + 42) = x^4 (x + 6) \times (x + 7)$$

The polynomial above was already factored in exercise 16 part c)

$$(x^6 + 13x^5 + 42x^4) / x^3 (x + 6) = \frac{x^4 (x + 6) \times (x + 7)}{x^3 (x + 6)}$$

$$= \frac{x^4 (\cancel{x} \cancel{6}) \times (x + 7)}{x^3 (\cancel{x} \cancel{6})}$$

$$= \frac{x^4 \times (x + 7)}{x^3}$$

$$= \frac{x^3 \times x \times (x + 7)}{x^3}$$

$$= \frac{\cancel{x}^3 \times x \times (x + 7)}{\cancel{x}^3}$$

$$= x \times (x + 7)$$

23.

Solve $x^2 = -3x + 10$ using the quadratic formula

Put in standard form

$$-10 + 3x + x^2 = -3x + 3x + 10 + -10$$

$$x^2 + 3x + -10 = 0$$

Use the quadratic formula. Compare $x^2 + 3x + -10 = 0$ with the standard form $ax^2 + bx + c = 0$ and you will see that $a = 1$, $b = 3$, and $c = -10$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 1 \times -10}}{2 \times 1}$$

$$x = \frac{-3 \pm \sqrt{9 - 4 \times 1 \times -10}}{2 \times 1}$$

$$x = \frac{-3 \pm \sqrt{9 + -4 \times 1 \times -10}}{2 \times 1}$$

$$x = \frac{-3 \pm \sqrt{9 + -4 \times -10}}{2 \times 1}$$

$$x = \frac{-3 \pm \sqrt{9 + 40}}{2 \times 1}$$

$$x = \frac{-3 \pm \sqrt{49}}{2 \times 1}$$

$$x = \frac{-3 \pm 7}{2 \times 1}$$

$$x = \frac{-3 \pm 7}{2}$$

$$x_1 = \frac{-3 + 7}{2} = \frac{4}{2} = 2$$

$$x_2 = \frac{-3 - 7}{2} = -\frac{10}{2} = -5$$

Now solve by completing the square

$$x^2 + 3x + -10 = 0$$

$$x^2 + 3x + -10 + 10 = 0 + 10$$

$$x^2 + 3x = 10$$

To complete the square, take half of b that is 3, square it, and add it to both sides of the equation.

$$\text{Half of 3 is } \frac{3}{2} \quad \text{Squaring this gives } \left(\frac{3}{2}\right)^2$$

$$x^2 + 3x + \left(\frac{3}{2}\right)^2 = 10 + \left(\frac{3}{2}\right)^2$$

$$\left(x + \frac{3}{2}\right)^2 = 10 + \frac{9}{4}$$

$$\left(x + \frac{3}{2}\right)^2 = \frac{10}{1} + \frac{9}{4}$$

$$\left(x + \frac{3}{2}\right)^2 = \frac{10 \times 4}{1 \times 4} + \frac{9}{4}$$

$$\left(x + \frac{3}{2}\right)^2 = \frac{40}{4} + \frac{9}{4}$$

$$\left(x + \frac{3}{2}\right)^2 = \frac{49}{4}$$

$$\sqrt{\left(x + \frac{3}{2}\right)^2} = \sqrt{\frac{49}{4}}$$

$$x + \frac{3}{2} = \pm \frac{7}{2}$$

$$x + \frac{3}{2} = +\frac{7}{2} \quad \text{and} \quad x + \frac{3}{2} = -\frac{7}{2}$$

$$x + \frac{3}{2} - \frac{3}{2} = +\frac{7}{2} - \frac{3}{2}$$

$$x + \frac{3}{2} - \frac{3}{2} = +\frac{7}{2} - \frac{3}{2}$$

$$x = \frac{4}{2} = 2$$

$$x + \frac{3}{2} - \frac{3}{2} = -\frac{7}{2} - \frac{3}{2}$$

$$x = -\frac{10}{2} = -5$$

24.

$$x = 1.012555555555$$

The technique is to put the decimal point immediately before and immediately after the repeating digit(s)

Let's put it before. Since 5 is the repeating digit, we will put it immediately before 5. To put before, multiply both sides of the equation by 1000

$$\text{We get } 1000x = 1.01255555555 \times 1000$$

$$1000x = 1012.55555 \quad (1)$$

Let's put it after. To put it after, multiply both sides of the equations by 10000

$$\text{We get } 10000x = 1.0125555555 \times 10000$$

$$10000x = 10125.5555 \quad (2)$$

Subtract (1) from (2)

$$10000x - 1000x = 10125.55555 - 1012.55555$$

$$9000x = 9113$$

$$x = \frac{9113}{9000}$$

24. $3 - \sqrt{x-2} = 1$

$$3 - \sqrt{x-2} = 1$$

$$3 + -\sqrt{x-2} = 1$$

$$3 - 3 + -\sqrt{x-2} = 1 - 3$$

$$-\sqrt{x-2} = -2$$

$$\left(-\sqrt{x-2}\right)^2 = (-2)^2$$

$$\left(-\sqrt{x-2}\right)^2 = (-2)^2$$

$$x - 2 = 4$$

$$x - 2 + 2 = 4 + 2$$

$$x = 6$$

25.

$$2 / \sqrt{5}$$

$$\frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{2\sqrt{5}}{\sqrt{25}} = \frac{2\sqrt{5}}{5}$$

$$\text{b) } 4 / (\sqrt{5} - 2)$$

$$\frac{4}{\sqrt{5} - 2} = \frac{4 \times (\sqrt{5} + 2)}{(\sqrt{5} - 2) \times (\sqrt{5} + 2)}$$

$$= \frac{4 \times (\sqrt{5} + 2)}{(\sqrt{5} - 2) \times (\sqrt{5} + 2)}$$

$$= \frac{4 \times \sqrt{5} + 4 \times 2}{(\sqrt{5} \sqrt{5} + \sqrt{5} \times 2 + -2 \times \sqrt{5} + -2 \times 2)}$$

$$= \frac{4 \times \sqrt{5} + 8}{(\sqrt{25} + \sqrt{5} \cancel{\times 2} + -2 \cancel{\times \sqrt{5}} + -4)}$$

$$= \frac{4 \times \sqrt{5} + 8}{(\sqrt{25} + -4)}$$

$$= \frac{4 \times \sqrt{5} + 8}{5 + -4}$$

$$= \frac{4 \times \sqrt{5} + 8}{1}$$

$$= 4 \times \sqrt{5} + 8$$

27.

$$\tan (30^{\circ}) = \frac{x}{15}$$

$$\frac{\sqrt{3}}{3} = \frac{x}{15}$$

$$15 \times \frac{\sqrt{3}}{3} = \frac{x}{15} \times 15$$

$$5 \times \sqrt{3} = x$$

$$8.66 \text{ cm} = x$$

28.

Let $2x$ be the first even integer

Let $2x + 2$ be the second even integer.

Since the sum of these two is 18, we can write

$$2x + 2x + 2 = 18$$

$$4x + 2 = 18$$

$$4x + 2 - 2 = 18 - 2$$

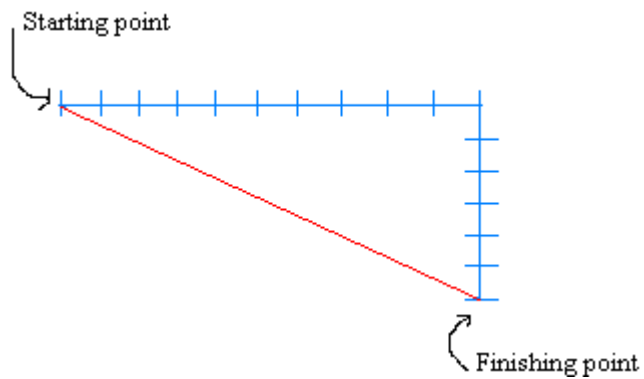
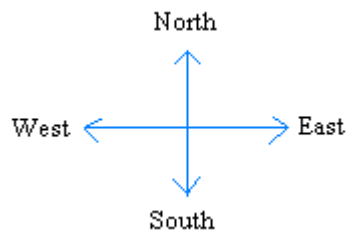
$$4x = 16$$

$$x = 4$$

The first even integer is $2x = 2 \times 4 = 8$

The second even integer is $2x + 2 = 2 \times 4 + 2 = 8 + 2 = 10$

29.



The distance that you are looking for is shown in red.

Use the Pythagorean Theorem to solve this problem. So c is the length that you are looking for.

$$c^2 = a^2 + b^2$$

$$c^2 = 10^2 + 6^2$$

$$c^2 = 100 + 36$$

$$c^2 = 136$$

$$c = 11.66 \cong 12$$

Therefore, you are almost 12 blocks away from your house

30.

Area = length $\times$ width

Area = $l \times w$

The length of a rectangle is two more than the width means:

Length = width + 2

Let $w = x$, $L = x + 2$

Area = 35.

Just replace these into Area = $l \times w$

$$35 = (x + 2) \times x$$

$$35 = x \times x + 2 \times x$$

$$35 = x^2 + 2x$$

$$x^2 + 2x = 35$$

$$x^2 + 2x + -35 = 35 + -35$$

$$x^2 + 2x + -35 = 0$$

Now factor the expression and set the factorization form equal to 0

To factor $x^2 + 2x + -35$ means that it will look like this $(x + \text{blank})(x + \text{blank})$

To fill in the blank, just look for factors of -35 that add up to 2.

$$-35 = -7 \times 5$$

$$-35 = 35 \times -1$$

$$-35 = 7 \times -5 \quad \text{since } 7 + -5 = 2, \text{ you will fill in the blank with 7 and -5}$$

$$-35 = -35 \times 1$$

$$x^2 + 2x + -35 = 0 \text{ is equivalent to } x^2 + 2x + -35 = (x + 7)(x + -5) = 0$$

$$x + 7 = 0$$

$$x + 7 - 7 = 0 - 7$$

$$x = -7$$

$$x + -5 = 0$$

$$x + -5 + 5 = 0 + 5$$

$$x = 5$$