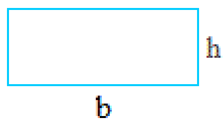
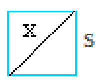
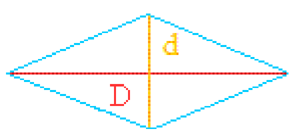
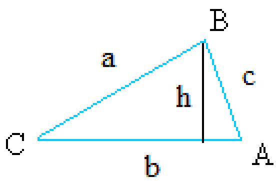
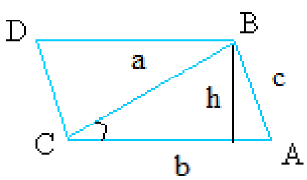
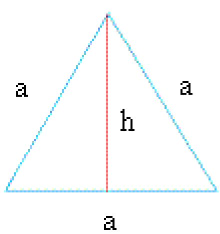
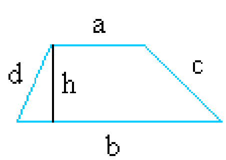
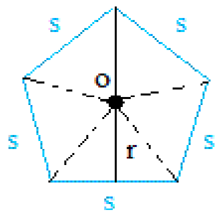
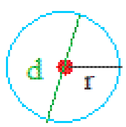
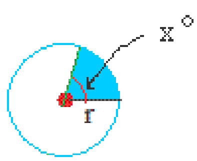
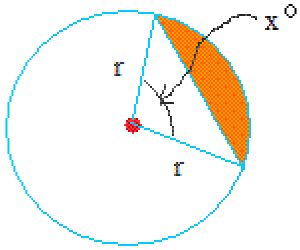
Geometric Formulas

Area and perimeter Formulas

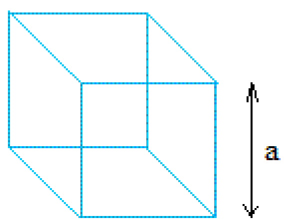
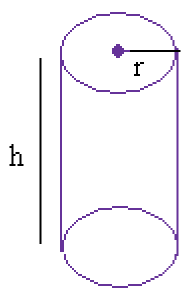
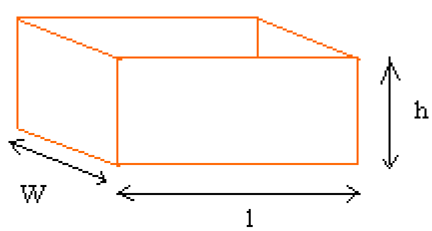
Rectangle:  	Perimeter = 2 × b + 2 × h Area = b × h
Square  	Perimeter = s + s + s + s = 4 × s $\text{Area} = s^2 = s \times s$ $\text{Area} = \frac{x^2}{2}$ x: length of diagonal
Rhombus  	$\text{Area} = \frac{d \times D}{2}$

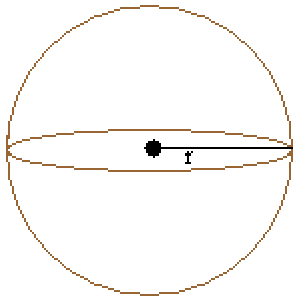
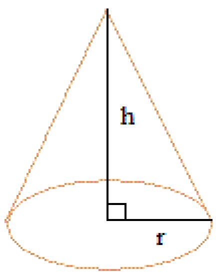
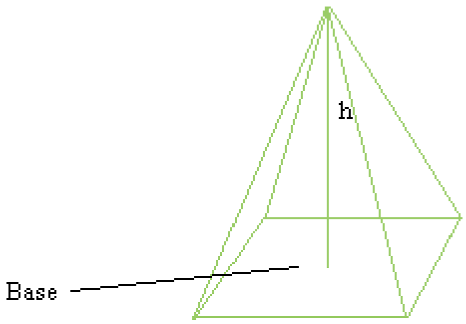
Triangle 	Perimeter = a + b + c $\text{Area} = \dfrac{b \times h}{2}$ $\text{Area} = \dfrac{a \times b \times \sin C}{2}$ $\text{Area} = \sqrt{s \times (s-a) \times (s-b) \times (s-c)}$ $s = \frac{1}{2}(a + b + c)$
Parallelogram 	Perimeter = $2 \times b + 2 \times c$ Area = $b \times h$ Area = $a \times b \times \sin C$
Equilateral triangle 	$A = \dfrac{a^2 \times \sqrt{3}}{4}$ $A = \dfrac{h^2 \times \sqrt{3}}{3}$
Trapezoid 	Perimeter = a + b + c + d $\text{Area} = \dfrac{a + b}{2} \times h$

Representation of a regular polygon or n-gon  	$\text{Perimeter} = n \times s$ $\text{Area} = \dfrac{r \times p}{2}$ p : perimeter r : apothem $\text{Area} = \dfrac{r \times n \times s}{2}$
Circle  	$\text{Perimeter} = 2 \times \pi \times r$ $\text{Area} = p \times r^2$ $\text{Area} = \dfrac{\pi \times d^2}{4}$
Sector  	$\text{Area} = \pi \times r^2 \times \left(\dfrac{x^\circ}{360}\right)$

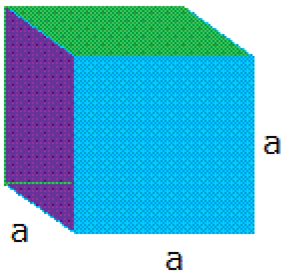
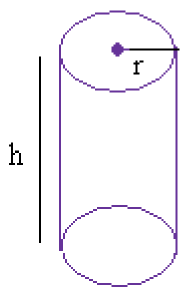
Minor segment 	Minor segment is in orange Area = Area of sector - area of triangle $\text{Area} = \pi \times r^2 \times \left(\frac{x^\circ}{360}\right) - \frac{r \times r \times \sin x^\circ}{2}$

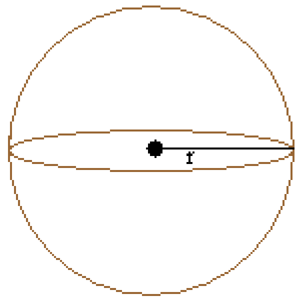
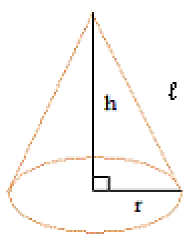
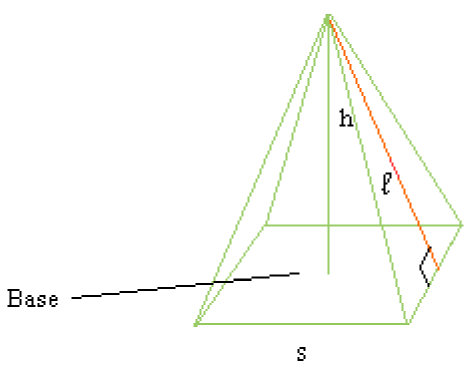
Volume Formulas

Cube  	$v = a^3 = a \times a \times a$
Cylinder  	$v = \pi \times r^2 \times h$ $\pi = 3.14$ h is the height r is the radius
Rectangular solid:  	Volume = l × w × h l is the length w is the width h is the height

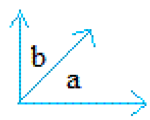
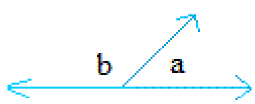
Sphere: 	$\text{Volume} = \frac{4 \times \pi \times r^3}{3}$ r is the radius $\pi = 3.14$
Cone 	$\text{Volume} = \frac{\pi \times r^2 \times h}{3}$ $\pi = 3.14$ r is the radius h is the height
Pyramid 	$\text{Volume} = \frac{B \times h}{3}$ B is the area of the base h is the height

Surface area Formulas

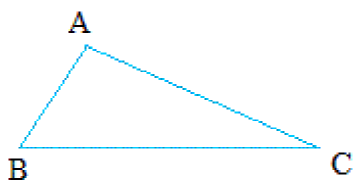
Cube 	$SA = 6a^2$
Cylinder 	$SA = 2 \times pi \times r^2 + 2 \times pi \times r \times h$ $\pi = 3.14$ h is the height r is the radius
Rectangular solid: 	$SA = 2 \times l \times w + 2 \times l \times h$ $+ 2 \times w \times h$

Sphere: 	$SA = 4 \times pi \times r^2$ r is the radius $\pi = 3.14$
Cone 	$SA = \pi r^2 + \pi r l$ $\pi = 3.14$ r is the radius h is the height
Pyramid 	$SA = s^2 + 2 \times s \times \ell$: s : length of the square base ℓ : Slant height of a triangle h: height of the pyramid

Angles Formulas

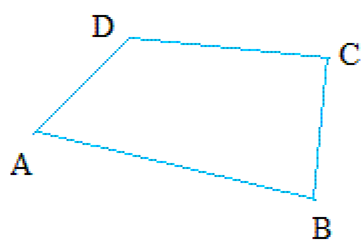
Complementary angles Complement of a° Complement of b° 	Complement of a°= 90° – b° Complement of b°= 90° – a°
Supplementary angles Supplement of a° Supplement of b° 	Supplement of a°= 90° – b° Supplement of b°= 90° – a°

Angle sum theorem: sum of the interior angles of a triangle.

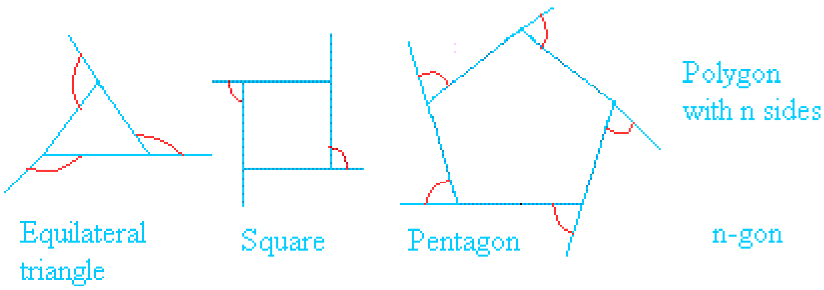
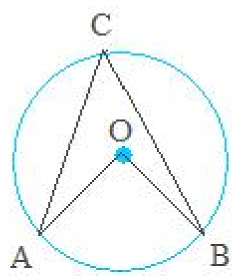


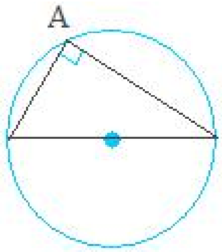
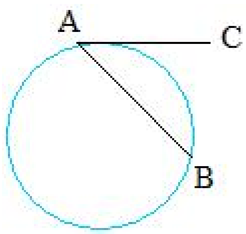
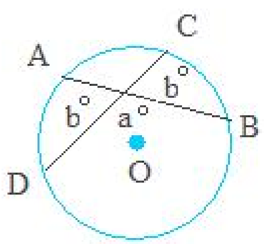
$m\langle A + m\langle B + m\langle C = 180°$

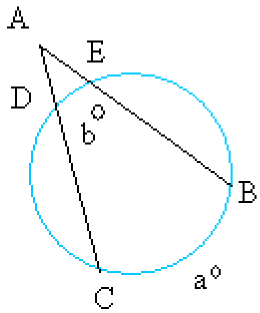
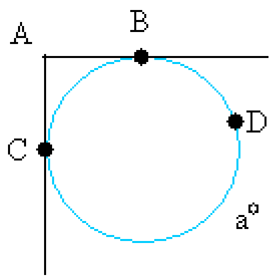
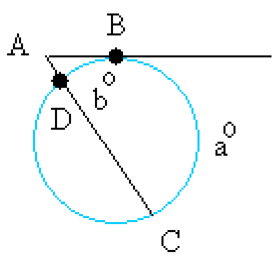
Sum of the interior angles of a quadrilateral

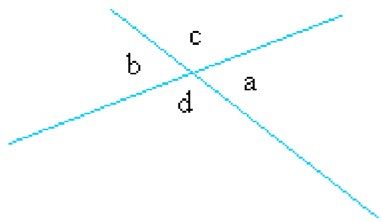
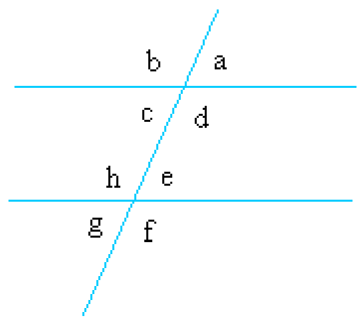
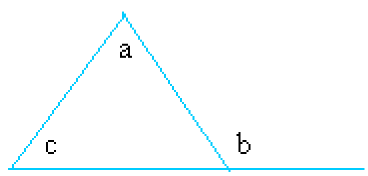


$m\langle A + m\langle B + m\langle C + m\langle D = 360°$

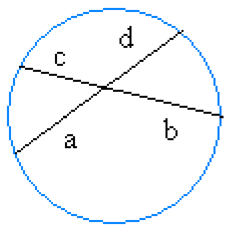
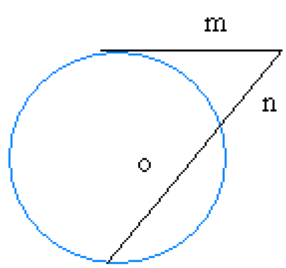
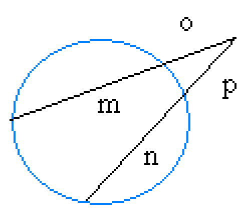
Measure of each exterior angle of a regular n-gon or a figure with n sides 	$\frac{360^\circ}{n}$
	m stands for measure ∠ AOB = Central angle ∠ ACB = Inscribed angle m∠ AOB = m (arc AB) m∠ ACB = ½ m (arc AB) m∠ ACB = ½ m∠ AOB

	Angle A is inscribed in a semi circle m∠ A = 90°
	m stands for measure ∠ BAC is formed by a tangent and a chord m∠ BAC = ½ m (arc AB)
	Angle formed by two intersecting chords m∠ b° = ½ (m arc CB + m arc AD) m∠ a° = ½ (m arc AC + m arc DB)

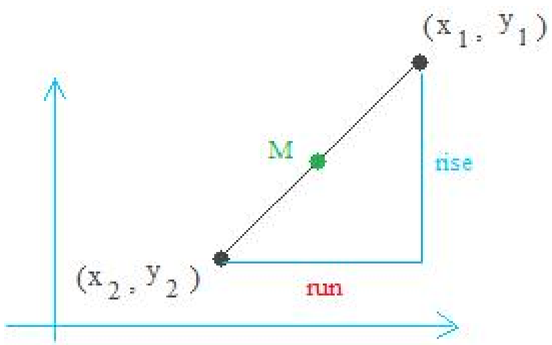
	$\angle A \doteq \frac{1}{2}\left(\overset{\frown}{BC} - \overset{\frown}{DE}\right)$ $m\angle A = \frac{1}{2}(a° - b°)$
	$\angle A \circeq \frac{1}{2}\left(\overset{\frown}{BDC} - \overset{\frown}{BC}\right)$ $m\angle A = \frac{1}{2}(a° - b°)$ $m\angle A = \frac{1}{2}(180° - b°)$
	$\angle A \doteq \frac{1}{2}\left(\overset{\frown}{BC} - \overset{\frown}{BD}\right)$ $m\angle A = \frac{1}{2}(a° - b°)$

Vertical angles 	$\angle a = \angle b$ $\angle c = \angle d$
	$\angle a = \angle c = \angle e = \angle g$ $\angle b = \angle d = \angle h = \angle f$
	$\angle b = \angle a + \angle c$

Other circle Formulas

Intersecting chords 	$\frac{a}{c} = \frac{b}{d}$ $a \times d = c \times b$
Intersecting tangent and secant 	$\frac{o}{m} = \frac{m}{n}$ $m^2 = o \times n$
Intersecting secants 	$m \times o = n \times p$

Coordinate geometry Formulas



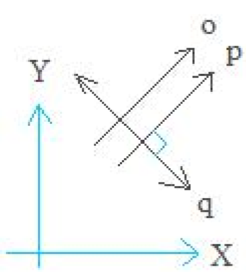
Slope = rise / run = rise ÷ run

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

$$\text{Midpoint} = M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

The distance formula is:

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

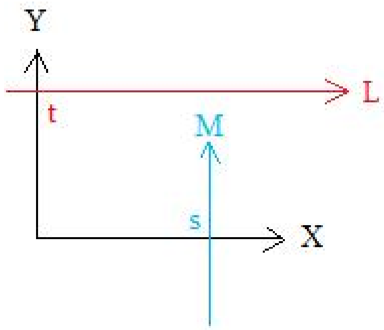
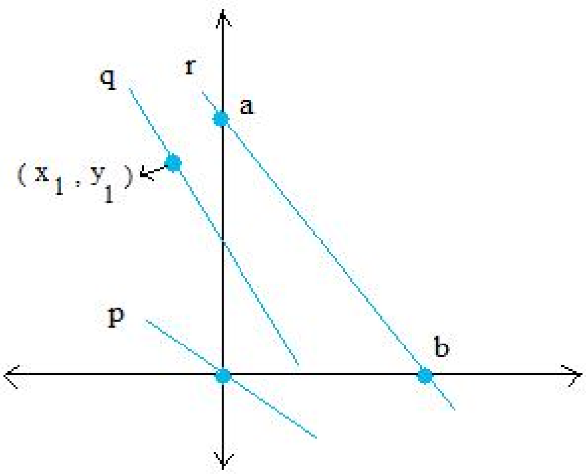


o and p are parallel, so they have the same slope m.

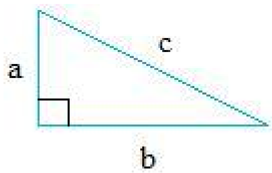
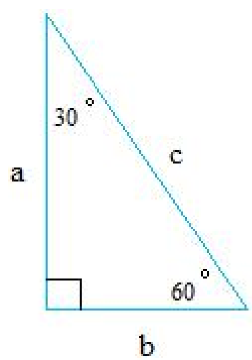
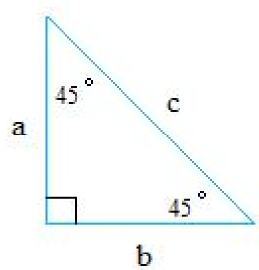
Let m be the slope for p
Let m’ be the slope for q

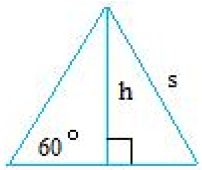
p and q are perpendicular, so the product of their slopes equals to -1

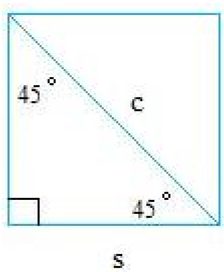
m × m’ = -1

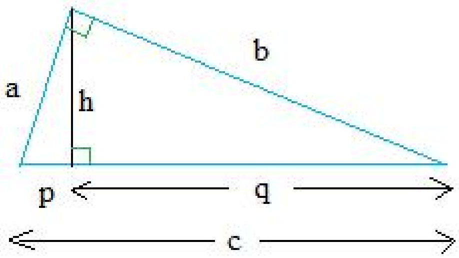
	Line L is parallel to x-axis: $y = t$, x is any number Line M is parallel to y-axis: $x = s$, y is any number
	Linear equations formulas Equation with slope m and y-intercept b $y = mx + b$ Line p: $y = mx$ since $b = 0$ Line q: slope-point form $y - y_1 = m(x - x_1)$ Line r: Equation with x-intercept b and y-intercept a. $\frac{y}{a} + \frac{x}{b} = 1$

Right triangle Formulas

	Pythagorean Theorem $c^2 = a^2 + b^2$ a and b are called legs c is the hypotenuse
Pythagorean Triples:	$(2m)^2 + \left(m^2 - 1\right)^2 = \left(m^2 + 1\right)^2$
	Leg opposite 30 ° angle: $b = \frac{c}{2}$ Leg opposite 60° angle: $a = \frac{c \times \sqrt{3}}{2}$ $a = b \times \sqrt{3}$ Leg opposite 45° angle: $a = \frac{c \times \sqrt{2}}{2}$ $a = b$

	Altitude or height of an equilateral triangle $h = \dfrac{s \times \sqrt{3}}{2}$ Side of an equilateral triangle. $s = \dfrac{2 \times h}{\sqrt{3}}$

	Side of a square: $s = \dfrac{c \times \sqrt{2}}{2}$ s: length of side Diagonal of a square: $c = s \times \sqrt{2}$ c: length of diagonal



$$\frac{p}{h} = \frac{h}{q} \qquad h^2 = p \times q \qquad h = \sqrt{p \times q}$$

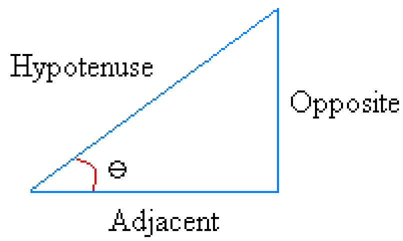
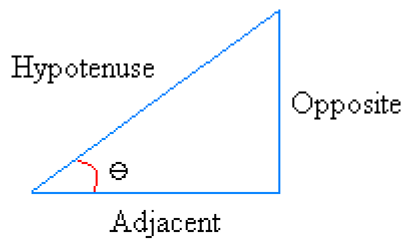
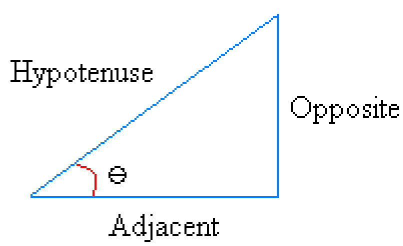
$$\frac{c}{a} = \frac{a}{p} \qquad a^2 = p \times c \qquad a = \sqrt{p \times c}$$

$$\frac{c}{b} = \frac{b}{q} \qquad b^2 = q \times c \qquad b = \sqrt{q \times c}$$

Total number of diagonals Formula

Total number of diagonals of an n-gon:	$\frac{n \times d}{2}$ n: number of sides d: number of diagonals from only one vertex

Basic trigonometric Formulas

	$\sin\theta = \dfrac{\text{opposite}}{\text{Hypotenuse}}$
	$\cos\theta = \dfrac{\text{Adjacent}}{\text{Hypotenuse}}$
	$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

Area and perimeter Formulas problems

Rectangle:

Word problem #1:

The surface of a table is a rectangle with width equal to 4 feet and length equal to 7 feet.
What is the perimeter of the table? What is the area?

Perimeter = $2 \times b + 2 \times h = 2 \times w + 2 \times L = 2 \times 4 + 2 \times 7 = 8 + 14 = 22$ feet
Area = $b \times h = 4 \times 7 = 28$ square feet.

Word problem #2:

The perimeter of a small room floor is 64 feet and the width is 8 feet. What is the length of the room?

Perimeter = $2 \times w + 2 \times L$

$48 = 2 \times 8 + 2 \times L$

$48 = 16 + 2 \times L$

$48 - 16 = 16 - 16 + 2 \times L$

$32 = 2 \times L$

L = 16 feet since 16 times 2 = 32.

Word problem #3:

The area of a notebook is 12 square inches and the height is 4 inch. What is the width?

Area = $b \times h = w \times h$

$12 = w \times h$

$12 = 4 \times h$

So, h = 3 since 4 times 3 = 12

Square

Word problem #4:

The length of one side of a square is 2 inches. What is the area of the square?

$$\text{Area} = s \times s = 2 \times 2 = 4 \text{ square inches}$$

The length of the diagonal of a square is $2 \times \sqrt{2}$.What is the area of the square?

$$\text{Area} = \frac{x^2}{2} = \frac{\left(2\sqrt{2}\right)^2}{2} = \frac{2\sqrt{2} \times 2\sqrt{2}}{2} = \frac{4 \times \sqrt{2} \times \sqrt{2}}{2}$$

$$= \frac{4 \times \sqrt{2 \times 2}}{2} = \frac{4 \times \sqrt{4}}{2} = \frac{4 \times 2}{2} = \frac{8}{2} = 4$$

Word problem #5:

The area of a square is 64 square inches, what is the length of a side?

$A = s \times s$

64 = $s \times s$. Since 8 times 8 equals 64, s = 8.

Word problem #6:
:
The area of a square is 4 square inches, what is the length of a diagonal?

$\text{Area} = \frac{x^2}{2}$

Replace A by 4. So

$4 = \frac{x^2}{2}$

Multiply both sides by 2 to get

$$4 \times 2 = \frac{x^2}{2} \times 2$$

Then, $8 = x^2$

$$x = \sqrt{8} = \sqrt{4 \times 2} = \sqrt{4} \times \sqrt{2} = 2 \times \sqrt{2} = 2\sqrt{2}$$

Rhombus

Word problem #7:

The diagonals of a rhombus are 4 feet and 6 feet. What is the area of the rhombus?

$$\text{Area} = \frac{d \times D}{2} = \frac{4 \times 6}{2} = \frac{24}{2} = 12$$

Word problem #8:

The area of a rhombus is 12 square feet and the big diagonal is 6. How long is the other diagonal?

$$12 = \frac{d \times 6}{2}$$

Multiply both sides by 2 to get 24 = $d \times 6$. Since 6 times 4 equals 24, d = 4 feet

Triangle

Word problem #9:

A triangle has sides equal to 2, 5, and 6 inches. What is the perimeter?

P = 2 + 5 + 6 = 13

The perimeter of a triangle is 18. Two other sides have lengths equals to 7 and 5. What is the length of the third side?

7 + 5 + length of third side = 18.
12 + length of third side = 18
Since 12 plus 6 = 18, the length of the third side is 6.

Note: The procedure to find the area of a triangle or the length or width using the formula below:

$$A = \frac{b \times h}{2}$$

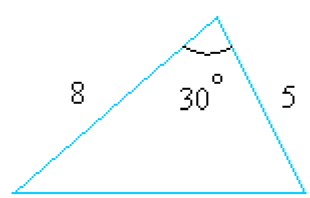
is very similar to finding the area or diagonals of a rhombus, so it will omitted here to avoid unnecessary repetition.

However, we shall show how to use the formula

$$\text{Area} = \frac{a \times b \times \sin C}{2}$$

Word problem #10

Find the area of the shape below with sides 8 and 5 inches and the included angle is $30°$



$$\text{Area} = \frac{a \times b \times \sin C}{2}$$

$$\text{Area} = \frac{8 \times 5 \times \sin(30)°}{2}$$

$$\text{Area} = \frac{40 \times \sin(30°)}{2}$$

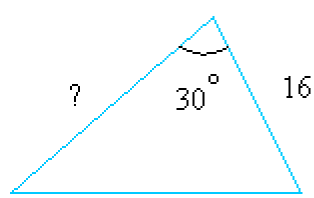
$$\text{Area} = \frac{40 \times 0.5}{2}$$

$$\text{Area} = \frac{20}{2}$$

$$\text{Area} = 10 \text{ square inches}$$

Word problem #11

If the area is 40 square inches, what is the length of the side shown with a question mark?



$$\text{Area} = \frac{a \times b \times \sin C}{2}$$

$$40 = \frac{a \times 16 \times \sin 30°}{2}$$

$$40 = \frac{a \times 16 \times 0.5}{2}$$

$$40 = \frac{a \times 8}{2}$$

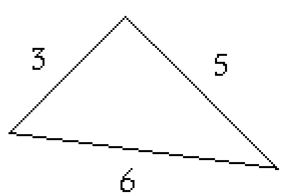
$$40 \times 2 = \frac{a \times 8}{2} \times 2$$

$$80 = a \times 8$$

Since 10 times 8 equals 80, a = 10 inches.

Word problem #12

What is the area of the triangle below?



$$\text{Area} = \sqrt{s \times (s-a) \times (s-b) \times (s-c)}$$

s = ½ (a + b + c)

s = ½ (3 + 6 + 5)

s = ½ (14)

s = 7

$$\text{Area} = \sqrt{s \times (s-a) \times (s-b) \times (s-c)}$$

$$\text{Area} = \sqrt{7 \times (7-3) \times (7-6) \times (7-5)}$$

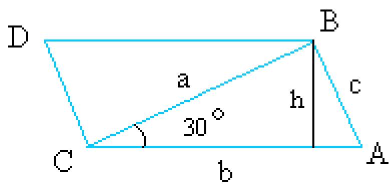
$$\text{Area} = \sqrt{7 \times (4) \times (1) \times (2)}$$

$$\text{Area} = \sqrt{56}$$

Area = 7.48

Parallelogram

Word problem #13



Find the perimeter and area when a = 6, c = 4, b = 8, and h = 3 inches

Use the following formula to check that you will get the same answer for the area.

$\text{Area} = a \times b \times \sin C$

$\text{Perimeter} = 2 \times b + 2 \times c$

$\text{Perimeter} = 2 \times 8 + 2 \times 4$

$\text{Perimeter} = 16 + 8 = 24$ inches

$\text{Area} = b \times h = 8 \times 3 = 24$ square inches

$\text{Area} = a \times b \times \sin C$

$\text{Area} = 6 \times 8 \times \sin(30°)$

$\text{Area} = 6 \times 8 \times 0.5$

$\text{Area} = 6 \times 4$

$\text{Area} = 24$ square inches

Equilateral triangle

Word problem #14

One side of an equilateral triangle equals 6 inches. What is the area?

$$A = \frac{a^2 \times \sqrt{3}}{4}$$

$$A = \frac{6^2 \times \sqrt{3}}{4}$$

$$A = \frac{36 \times \sqrt{3}}{4}$$

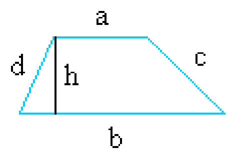
$$A = \frac{36}{4} \times \sqrt{3}$$

$$A = 9 \times \sqrt{3}$$

Trapezoid

Word problem #15

In the trapezoid below, a, b, c, d, and h are equal to 4, 8, 5, 3, and 2 inches respectively. Find the perimeter and the area of the trapezoid



Perimeter = a + b + c + d = 4 + 8 + 5 + 3

Perimeter = 12 + 8 = 20 inches

$$\text{Area} = \frac{a+b}{2} \times h$$

$$\text{Area} = \frac{4+8}{2} \times 2$$

$$\text{Area} = \frac{12}{2} \times 2$$

$$\text{Area} = 6 \times 2$$

Area = 12 inches

Word problem #16

If the area of a trapezoid is 18 square inches, the height is 4, and one of the parallel sides is 6 inches. What is the length of the other parallel side?

$$\text{Area} = \frac{a+b}{2} \times h$$

$$18 = \frac{6+b}{2} \times 4$$

$$2 \times 18 = 2 \times \frac{6+b}{2} \times 4$$

$$2 \times 18 = \cancel{2} \times \frac{6 + b}{\cancel{2}} \times 4$$

$$2 \times 18 = (6 + b) \times 4$$

$$36 = (6 + b) \times 4$$

$$36 = 24 + 4b$$

$$36 - 24 = 24 - 24 + 4b$$

$$12 = 4b$$

$$\frac{12}{4} = \frac{4}{4} b$$

$$3 = b$$

Regular n-gon

Word problem #17

Find the perimeter and area of a decagon if one side of the decagon is equal to 4 inches and the apothem is 3 inches.

$$p = n \times s$$

$$\text{Area} = \frac{r \times p}{2}$$

s = 4 and r = 3

Since a decagon has 10 sides, n = 10

$$p = 10 \times 4 \text{ inches}$$

p = 40 inches

$$\text{Area} = \frac{r \times p}{2}$$

$$\text{Area} = \frac{3 \times 40}{2}$$

$$\text{Area} = \frac{120}{2}$$

Area = 60 square inches

Word problem #18

The area of a hexagon is 36 square inches. The apothem is 12 inches.
Find the length of one side of the octagon.

$$36 = \frac{12 \times p}{2}$$

$$36 \times 2 = 2 \times \frac{12 \times p}{2}$$

$$72 = 12 \times p$$

$$\frac{72}{12} = \frac{12}{12} \times p$$

p = 6

Now use, $p = n \times s$

$$6 = 6 \times s$$

So s = 1

Circle problems

Word problem #19

The base of a swimming pool is shaped like a circle and has a radius of 10 feet. What is the perimeter and area of the base?

$$\text{Perimeter} = 2 \times \pi \times r$$
$$\text{Perimeter} = 2 \times 3.14 \times 10$$
$$\text{Perimeter} = 6.28 \times 10$$
$$\text{Perimeter} = 6.28 \times 10$$
$$\text{Perimeter} = 62.8 \text{ feet}$$

$$\text{Area} = \pi \times r^2$$
$$\text{Area} = 3.14 \times 10^2$$
$$\text{Area} = 3.14 \times 100$$
$$\text{Area} = 314 \text{ square feet}$$

You could also use the formula below

$$\text{Area} = \frac{\pi \times d^2}{4}$$

$$\text{Area} = \frac{3.14 \times 20^2}{4}$$

$$\text{Area} = \frac{3.14 \times 400}{4}$$

$$\text{Area} = \frac{3.14 \times 4 \times 100}{4}$$

$$\text{Area} = \frac{12.56 \times 100}{4}$$

$$\text{Area} = \frac{1256}{4}$$

$$\text{Area} = 314 \text{ square feet}$$

Word problem #20

The base of a swimming pool is shaped like a circle and has an area of 400 square feet. How long is the diameter of the base? How long is the radius?

$$\text{Area} = \frac{\pi \times d^2}{4}$$

$$400 = \frac{3.14 \times d^2}{4}$$

$$4 \times 400 = \frac{3.14 \times d^2}{4} \times 4$$

$$4 \times 400 = \frac{3.14 \times d^2}{\cancel{4}} \times \cancel{4}$$

$$1600 = 3.14 \times d^2$$

$$\frac{1600}{3.14} = \frac{3.14}{3.14} \times d^2$$

$$509.55 = d^2$$

$$\sqrt{509.55} = \sqrt{d^2}$$

$$22.57 = d$$

The diameter is then 22.57 feet long. The radius is half the diameter, so the radius is 11.28 feet

Word problem #21

What is the area of a sector with an angle of 180 degrees and a radius of 6 inches?

$$\text{Area} = \pi \times r^2 \times \left(\frac{x^\circ}{360}\right)$$

$$\text{Area} = 3.14 \times 6^2 \times \frac{180}{360}$$

$$\text{Area} = 3.14 \times 36 \times \frac{180}{360}$$

$$\text{Area} = 3.14 \times 36 \times \frac{1}{2}$$

$$\text{Area} = 3.14 \times 18$$

Area = 56.52 square inches

Word problem #22

What is the angle of a sector with a radius of 10 inches and an area of 39.25 square inches?

$$\text{Area} = \pi \times r^2 \times \left(\frac{x^\circ}{360}\right)$$

$$39.25 = 3.14 \times 10^2 \times \frac{x^\circ}{360}$$

$$39.25 = 3.14 \times 100 \times \frac{x^\circ}{360}$$

$$39.25 = 314 \times \frac{x^\circ}{360}$$

$$360 \times 40 = 314 \times \frac{x°}{360} \times 360$$

$$360 \times 39.25 = 314 \times \frac{x°}{3\cancel{6}0} \times 3\cancel{6}0$$

$$360 \times 39.25 = 314 \times x°$$

$$14130 = 314 \times x°$$

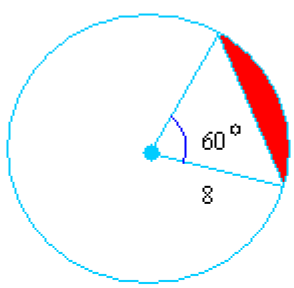
$$\frac{14130}{314} = \frac{314}{314} \times x°$$

$$45 = x°$$

The sector should then be 45 degrees

Word problem #23

What is the area of the minor segment in red if the central angle is 60 degrees and the radius of the circle is 8 inches?



$$\text{Area} = \pi \times r^2 \times \left(\frac{x°}{360}\right) - \frac{r \times r \times \sin x°}{2}$$

$$\text{Area} = 3.14 \times 8^2 \times \frac{60}{360} - \frac{8 \times 8 \times \sin(60°)}{2}$$

$$\text{Area} = 3.14 \times 64 \times \frac{60}{360} - \frac{64 \times \sin(60°)}{2}$$

$$\text{Area} = 3.14 \times 64 \times \frac{1}{6} - \frac{64 \times \sin(60°)}{2}$$ To get $\frac{1}{6}$,divide 60 by 60 and 360 by 60

$$\text{Area} = 200.96 \times \frac{1}{6} - \frac{64 \times \sin(60°)}{2}$$

$$\text{Area} = \frac{200.96}{1} \times \frac{1}{6} - \frac{64 \times \sin(60°)}{2}$$

$$\text{Area} = \frac{200.96 \times 1}{1 \times 6} - \frac{64 \times \sin(60°)}{2}$$

$$\text{Area} = \frac{200.96}{6} - \frac{64 \times \sin(60°)}{2}$$

$$\text{Area} = \frac{200.96}{6} - \frac{64 \times 0.86}{2}$$

$$\text{Area} = \frac{200.96}{6} - \frac{55.04}{2}$$

$$\text{Area} = 33.493 - 27.52$$

$$\text{Area} = 5.973 \approx 6$$

Volume problems

Cube

Word problem #1:

Find the volume if the length of one side is 2 cm

$V_{cube} = 2^3$

$V_{cube} = 2 \times 2 \times 2$

$V_{cube} = 8\ cm^3$

Word problem #2:

Find the length of a side of a cube if the volume is 27 cubic inches

$V = a \times a \times a$

$27 = a \times a \times a$

Since $3\times3\times3 = 27$, b $a = 3$ inches

Rectangular prism or box

Word problem #3:

An LCD tv was put in a box with a length of 2 feet, a height of 3 feet, and a width of 0.5 foot. What is the volume of the box?

$V_{box} = l \times w \times h$

$V_{box} = 2 \times 0.5 \times 3$

$V_{box} = 1\ ft^2 \times 3\ ft$

$V_{box} = 3\ ft^3$

Word problem #4:

The volume of a box is 24 cubic inches, the height is 3 inches and the length is 2 inches. What is the width of the box?

$V_{box} = l \times w \times h$

$24 = 2 \times w \times 3$

$24 = 6 \times w$.

Since 6 times 4 equals 24, the width is 4 inches

Sphere

Word problem #5:

Find V_{sphere} if r = 2 inches

$V_{sphere} = \frac{4}{3} \times \pi \times r^3$

$V_{sphere} = \frac{4}{3} \times 3.14 \times 2^3$

$V_{sphere} = \frac{4}{3} \times 3.14 \times 8$

$V_{sphere} = \frac{4}{3} \times 25.12$

$V_{sphere} = \frac{4}{3} \times \frac{25.12}{1}$

$V_{sphere} = \frac{4 \times 25.12}{3 \times 1}$

$V_{sphere} = \frac{100.48}{3}$

$V_{sphere} = 33.49 \text{ inches}^3$

Word problem #6:

A ball has a volume of 60 cubic meters. What is the radius of the ball?

$V_{sphere} = \frac{4}{3} \times \pi \times r^3$

$60 = \frac{4}{3} \times \pi \times r^3$

$60 = \frac{4}{3} \times 3.14 \times r^3$

$60 = \frac{4}{3} \times \frac{3.14}{1} \times r^3$

$$60 = \frac{12.56}{3} \times r^3$$

$$60 = \frac{12.56}{3} \times r^3$$

$60 = 4.1866 \times r^3$

$$\frac{60}{4.1866} = \frac{4.1866}{4.1866} \times r^3$$

$14.331 = r^3$

$$\sqrt[3]{14.331} = \sqrt[3]{r^3}$$

$2.428 = r$

The radius of the ball is 2.428 inches.

Cylinder

Word problem #7:

Calculate Volume   if r = 2 cm and h = 5 cm

Volume = $\pi \times r^2 \times h$

Volume = 3.14 × 22× 5

Volume = 3.14 × 4 × 5

Volume = 3.14 × 20

Volume = 62.8 cm^3

Word problem #8:

The volume of a cylinder is 60 cm^3 and the height of the cylinder is 7 cm. What is the radius of the cylinder?

Volume = $\pi \times r^2 \times h$
$60 = 3.14 \times r^2 \times 7$
$60 = 21.98 \times r^2$

$$\frac{60}{21.98} = \frac{21.98}{21.98} \times r^2$$

$2.729 = r^2$

$\sqrt{2.729} = r^2$

$1.65 = r$

The radius of this cylinder is 1.65 cm

Cone

Word problem #9:

Calculate the volume if r = 2 cm and h = 3 cm

$V_{cone} = \frac{1}{3} \times \pi \times r^2 \times h$

$V_{cone} = \frac{1}{3} \times 3.14 \times 2^2 \times 3$

$V_{cone} = \frac{1}{3} \times 3.14 \times 4 \times 3$

$V_{cone} = \frac{1}{3} \times 3.14 \times 12$

$V_{cone} = \frac{1}{3} \times 37.68$

$V_{cone} = \frac{1}{3} \times \frac{37.68}{1}$

$V_{cone} = \frac{1 \times 37.68}{3 \times 1}$

$V_{cone} = \frac{37.68}{3}$

$V_{cone} = 12.56 \text{ cm}^3$

Word problem #10:

A cone has a volume of 50 cm^3 and a radius of 3 cm. What is the height of the cone?

$V_{cone} = \frac{1}{3} \times \pi \times r^2 \times h$

$$50 = \frac{1}{3} \times 3.14 \times 3^2 \times h$$

$$50 = \frac{1}{3} \times 3.14 \times 9 \times h$$

$$50 = \frac{1}{3} \times 3.14 \times \frac{9}{1} \times h$$

$$50 = \frac{1 \times 9}{3 \times 1} \times 3.14 \times h$$

$$50 = \frac{9}{3} \times 3.14 \times h$$

$$50 = 3 \times 3.14 \times h$$

$$50 = 9.42 \times h$$

$$\frac{50}{9.42} = \frac{9.42}{9.42} \times h$$

$$5.307 = h$$

Pyramid

Word problem #11:

A square pyramid has a height of 9 meters. If a side of the base measures 4 meters, what is the volume of the pyramid?

Volume of the pyramid = $\dfrac{B \times h}{3}$

Since the base is a square, area of the base = 4 × 4 = 16 m^2

Volume of the pyramid = $\dfrac{16 \times 9}{3}$ = 144/3 = 48 m^3

Word problem #12:

The volume of a triangular pyramid is 27 cm^3. The area of the base 8 cm^2 What is the height of the pyramid?

Volume of the pyramid = $\dfrac{B \times h}{3}$

$27 = \dfrac{8 \times h}{3}$

$27 \times 3 = \dfrac{8 \times h}{3} \times 3$

$81 = 8 \times h$

$\dfrac{81}{8} = h$

$10.125 = h$

The height of this pyramid is 10.125 cm

Surface area problems

Cube

Word problem #1:

Find the surface area if the length of one side is 3 cm

Surface area = $6 \times a^2$

Surface area = 6×3^2

Surface area = $6 \times 3 \times 3$

Surface area = 54 cm^2

Word problem #2:

The surface area of a cube is 96 cm^2. What is the length of a side of a square?

$96 = 6 \times a^2$

$$\frac{96}{6} = a^2$$

$16 = a^2$

Since $4^2 = 4$ times $4 = 16$, $a = 4$ cm

Box

Word problem #3:

Find the surface area of a rectangular prism with a length of 6 cm, a width of 4 cm, and a height of 2 cm

$SA = 2 \times l \times w + 2 \times l \times h + 2 \times w \times h$

$SA = 2 \times 6 \times 4 + 2 \times 6 \times 2 + 2 \times 4 \times 2$

$SA = 48 + 24 + 16$

$SA = 88 \text{ cm}^2$

Word problem #4:

A rectangular prism has a surface area of 52 cm^2. The height is 4 cm and the width is 2 cm. What is the length of the prism?

$SA = 2 \times l \times w + 2 \times l \times h + 2 \times w \times h$

$52 = 2 \times l \times 2 + 2 \times l \times 4 + 2 \times 2 \times 4$

$52 = 4 \times l + 8 \times l + 16$

$52 = 12 \times l + 16$

$52 - 16 = 12 \times l + 16 - 16$

$36 = 12 \times l$

Since 12 times 3 = 36, l = 3.

The length of this rectangular prism is 3 cm

Cylinder

Word problem #5:

Find the surface area of a cylinder with a radius of 2 cm, and a height of 1 cm

$SA = 2 \times \pi \times r^2 + 2 \times \pi \times r \times h$

$SA = 2 \times 3.14 \times 2^2 + 2 \times 3.14 \times 2 \times 1$

$SA = 6.28 \times 4 + 6.28 \times 2$

$SA = 25.12 + 12.56$

Surface area = 37.68 cm^2

Word problem #6:

The surface area of a cylinder is 200π cm^2 and the radius is 5 cm. What is the height of the cylinder?

$SA = 2 \times \pi \times r^2 + 2 \times \pi \times r \times h$

$200\pi = 2 \times \pi \times r^2 + 2 \times \pi \times r \times h$

$200\pi = 2 \times \pi \times 5^2 + 2 \times \pi \times 5 \times h$

$$\frac{200\pi}{2\pi} = \frac{2 \times \pi \times 5^2}{2\pi} + \frac{2\pi \times 5 \times h}{2\pi}$$

$100 = 5^2 + 5 \times h$

$100 = 25 + 5 \times h$

$100 - 25 = 25 - 25 + 5 \times h$

$75 = 5 \times h$

Since 5 times 15 = 75, h = 15 cm

Word problem #7
The surface area of a cylinder is 70π cm^2 and the height is 2 cm. What is the radius of the cylinder?

$$SA = 2 \times \pi \times r^2 + 2 \times \pi \times r \times h$$

$$70\pi = 2 \times \pi \times r^2 + 2 \times \pi \times r \times 2$$

$$\frac{70\pi}{2\pi} = \frac{2 \times \pi \times r^2}{2\pi} + \frac{2\pi \times r \times 2}{2\pi}$$

$$35 = r^2 + 2 \times r$$

$$35 - 35 = r^2 + 2 \times r - 35$$

$$r^2 + 2 \times r - 35 = 0$$

This is a quadratic equation.

To solve it, look for factors of –35 that will add up to the coefficient of r that is 2

$-35 = -7 \times 5$
$-35 = 7 \times -5$ and $7 + -5 = 2$

$-35 = 35 \times -1$
$-35 = -35 \times 1$

$$r^2 + 2 \times r - 35 = 0$$

$$(r + -5)(r + 7) = 0$$

$r + -5 = 0$ $r = 5$ This answer is valid since it is positive.

$r + 7 = 0$ $r = -7$

Square Pyramid

Word problem #8

Find the surface area with a base length of 3 cm, and a slant height of 2 cm

$SA = s^2 + 2 \times s \times \ell$

$SA = 3^2 + 2 \times 3 \times 2$

$SA = 9 + 12$

$SA = 21\ cm^2$

Word problem #9

The surface area of a square pyramid is 32 cm^2 and the length of one side of the base is 4 cm. What is the length of the slant height?

$SA = s^2 + 2 \times s \times \ell$

$32 = 4^2 + 2 \times 4 \times \ell$

$32 = 16 + 8 \times \ell$

$32 - 16 = 16 - 16 + 8 \times \ell$

$16 = 8 \times \ell$

Since 8 times 2 = 16, $\ell = 2$ cm

Sphere

Word problem #10

Find the surface area of a sphere with a radius of 6 cm

SA = $4 \times \pi \times r^2$

SA = $4 \times 3.14 \times 6^2$

SA = 12.56×36

SA = 452.16

Surface area = 452.16 cm^2

Word problem #11

The surface area of a sphere is 36π. What is the radius of the sphere?

SA = $4 \times \pi \times r^2$

$36\pi = 4 \times \pi \times r^2$

$$\frac{36\pi}{4\pi} = \frac{4 \times \pi \times r^2}{4\pi}$$

$9 = r^2$

$\sqrt{9} = r$

$3 = r$

Cone

Word problem #12

Find the surface area of a cone with a radius of 4 cm, and a height of 8 cm

$$S = \pi \times r^2 + \pi \times r \times \sqrt{r^2 + h^2}$$

$S = 3.14 \times 4^2 + 3.14 \times 4 \times \sqrt{4^2 + 8^2}$

$S = 3.14 \times 16 + 12.56 \times \sqrt{16 + 64}$

$S = 50.24 + 12.56 \times \sqrt{80}$

$S = 50.24 + 12.56 \times 8.94$

$S = 50.24 + 112.28$

$S = 162.52 \text{ cm}^2$

Word problem #13

The surface area of a cone is 100π and the radius of the base is 2. What is the height of the cone?

$$S = \pi \times r^2 + \pi \times r \times \sqrt{r^2 + h^2}$$

$$100\pi = \pi \times 2^2 + \pi \times 2 \times \sqrt{2^2 + h^2}$$

$$100\pi = \pi \times 4 + \pi \times 2 \times \sqrt{4 + h^2}$$

$$\frac{100\pi}{\pi} = \frac{\pi \times 4}{\pi} + \frac{\pi \times 2 \times \sqrt{4 + h^2}}{\pi}$$

$$100 = 4 + 2 \times \sqrt{4 + h^2}$$

$$100 - 4 = 4 - 4 + 2 \times \sqrt{4 + h^2}$$

$$96 = 2 \times \sqrt{4 + h^2}$$

$$\frac{96}{2} = \frac{2 \times \sqrt{4 + h^2}}{2}$$

$$48 = \sqrt{4 + h^2}$$

$$48^2 = \left(\sqrt{4 + h^2}\right)^2$$

$$2304 = 4 + h^2$$

$$2304 - 4 = h^2$$

$$2300 = h^2$$

$$h = \sqrt{2300}$$

$$h = 47.95$$

Angles problems

Complementary and supplementary angles.

Word problem #1

What is the complement of an angle whose is 30 degrees?
The complement of this angle is 90° – 30° = 60°

Word problem #2

The measure of one complementary angle is two times as big as the measure of the other. What is the measure of each angle?

Let x be the measure of one angle. Then, the measure of other angle is 2x.

Since the angles are complementary, they add up to 90 degrees.
Thus x + 2x = 90
3x = 90
x = 30
The other angle is 90° – 30° = 60°

Word problem #3

The measure of one supplementary angle is two times as big as the measure of the other. What is the measure of each angle?
Very similar problem except that this time, x + 2x = 180

Thus, 3x = 180
x = 60°
The other angle is 180° – 60° = 120°

Angle sum theorem

Word problem #4

Two angles of triangle measure 40 and 60 degrees respectively. What is the measure of the third angle?

Let x be the measure of the third angle.
$40 + 60 + x = 180$
$100 + x = 180$
Thus $x = 80°$

Angle sum in a quadrilateral

Word problem #5

Four angles of quadrilateral have the following conditions. The second angle is 20 more the first angle. The third is twice as big as the first one and the fourth is 50 less than the third

Let x be the measure of the first angle. Then, the second is x + 20. The third is 2x and the fourth is 2x - 50
Thus,

First		Second		Third		Fourth		
x	+	x + 20	+	2x	+	2x -50	=	360

$x + x + 2x + 2x + 20 - 50 = 360$
$6x - 30 = 360$
$6x - 30 + 30 = 360 + 30$
$6x = 390$
Since 6 times 65 = 390, $x = 65°$

Second angle is $65 + 20 = 85°$
Third angle is $2x = 2$ times $65 = 130°$

Fourth angle is 50 less than the third = 130 - 50 = 80°
Indeed 65° + 85° + 130° + 80° = 150° + 210° = 360°

n-gon

Word problem #6

What is the sum of the interior angles of an octagon or 8-gon?
The sum of the interior angles is given by 180° (n – 2)
Thus, 180° (n – 2) = 180° × (8 – 2) = 180° × (6) = 1080°

Word problem #7

The sum of he interior angles of a n-gon is 900. How many sides this n-gon have?
900 = 180° × (n – 2) Divide both sides by 180 to get the equation below:
5 = n - 2
Since 7 minus 2 equals 5, n = 7. The regular polygon is a heptagon or 7-gon.

Word problem #8

What is the measure of each interior angle of an octagon? What about the exterior angle?

Do everything you did in Word problem # 6 to get the sum of the interior angle, and then divide the answer by 8.
The interior angle is 1080 divided by 8 or 135°

There are two ways to get the exterior angle.
Use the formula below. n = number of sides. In this case n = 8

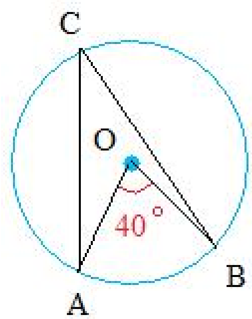
$$\frac{360^\circ}{n}$$

and do 360 divided by 8 to get 45°

Or do a close observation of the n-gon, and you will see that interior angle + exterior angle = 180°

So 135 + interior angle = 180
Since 135 + 45 = 180, exterior angle = 45°

Minor arc and major arc



Word problem #9

In the figure above, ∠ AOB = 40°. What is the length of major arc ACB? What is the measure of ∠ ACB ?

First, notice that three letters are used to for the major arc. However, for minor arc AB, you will use only 2 letters a shown.

m (arc AB) = m∠ AOB = 40°

m (arc AB) + m (arc ACB) = 360°

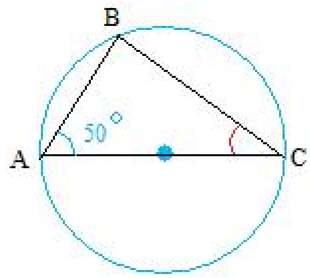
40° + m (arc ACB) = 360°

Since 40 + 320 = 360, measure of major arc ACB = 320°

m∠ ACB = ½ m∠ AOB

m∠ ACB = ½ of 40 = 20°

Word problem #10



What is the measure of $\angle C$?

Notice that the measure of arc AC is half the circle, so m (arc AC) = 180°

$m\angle B = \frac{1}{2}$ m (arc AC)

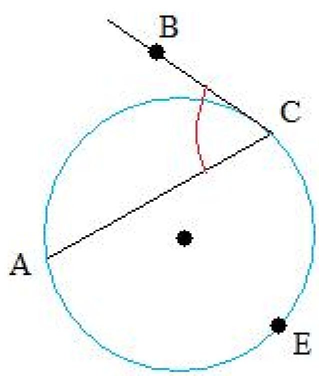
$m\angle ACB = \frac{1}{2}$ of 180° = 90°

So $90 + 50 + m\angle C = 180$

$140 + m\angle C = 180$

Since $140 + 40 = 180$, $m\angle C = 40°$

Word problem # 11

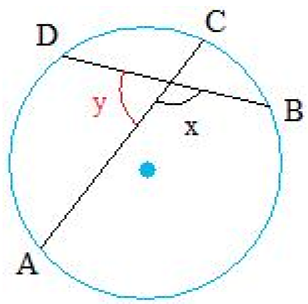


The measure of angle C shown in red is 60 degrees. What is the measure of the major arc AEC?

Measure of angle C is equal to half the measure of minor arc AC
m∠ C = ½ m∠ arc AC
60 = ½ of m∠ arc AC
Since ½ of 120 is equal to 60, m∠ arc AC = 120°
m (arc AC) + m (arc AEC) = 360°

120° + m (arc AEC) = 360°
Since 120 + 240 = 360, measure of major arc AEC = 240°

Word problem # 12



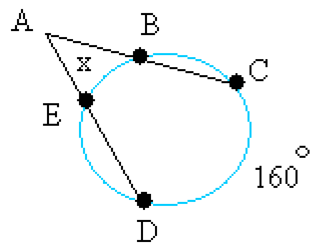
In the figure above, m (arc AB) = 180° and m (arc DC) = 30°
Find the measure of angle y.
First, find the measure of angle x.

m∠ x° = ½ (m arc AB + m arc DC)
= ½ (180 + 30)
= ½ (210)
= ½ of 210
= 105°

Now, since angle x and angle y make a straight line,
m ∠x + m∠y = 180°
105 + m ∠y = 180
Since 105 + 75 = 180, m∠y = 75°

Word problem # 13

If m $\angle A = 30°$, m (arc BE)



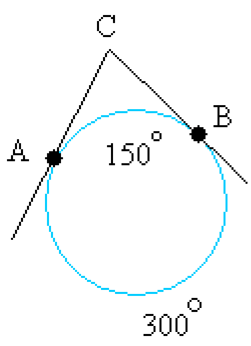
$$\angle A \doteq \frac{1}{2}\left(\overset{\frown}{DC} - \overset{\frown}{BE}\right)$$

$$\text{So, } 30 = \frac{1}{2}\left(160° - \overset{\frown}{BE}\right)$$

$$60 = 160° - \overset{\frown}{BE}$$

Since 160 – 100 = 60, arc BE = 100 °

Word problem # 14



m of major arc (AB) = 300 degrees
m of minor arc (AB) = 150 degrees
What is the measure of angle C?

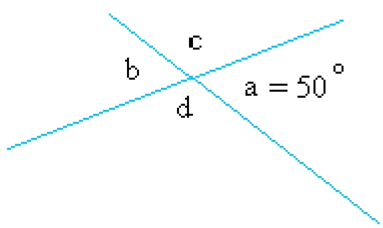
$$\angle C = \frac{1}{2}(300° - 150°)$$

$$\angle C = \frac{1}{2}(150°)$$

$$\angle C = 75°$$

Word problem # 15

Use the figure below to find angles b, c, and d



$\angle b = \angle a = 50°$

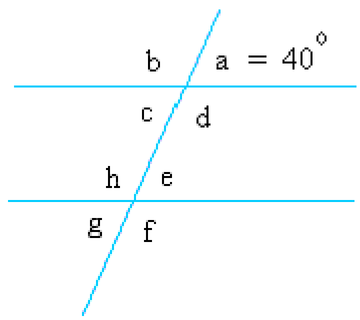
$\angle c + \angle a = 180°$

$\angle c + 50° = 180°$

Since $130 + 50 = 180$, $\angle c = 130°$

Word problem # 16

Use the figure below to find all other angles



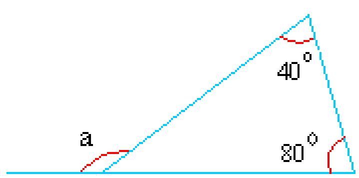
$\angle a = \angle c = \angle e = \angle g = 40°$

$\angle b = 140°$ since $140 + 40 = 180°$

$\angle b = \angle d = \angle h = \angle f = 140°$

Word problem # 17

Find the measure of the exterior angle a



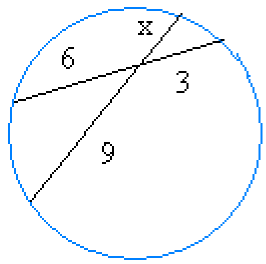
$\angle a = 40° + 80°$

$\angle a = 120°$

Other circle formula problems

Word problem # 1

Find the length of x



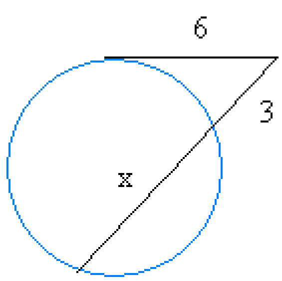
$9 \times x = 6 \times 3$

$9x = 18$

Since 9 times 2 = 18, x = 2

Word problem # 2

Find the length of chord x



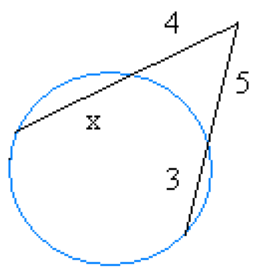
$6^2 = 3 \times x$

$36 = 3 \times x$

Since 3 times 12 = 36, x = 12

Word problem # 3

Find the length of chord x



$x \times 4 = 3 \times 5$

$x \times 4 = 15$

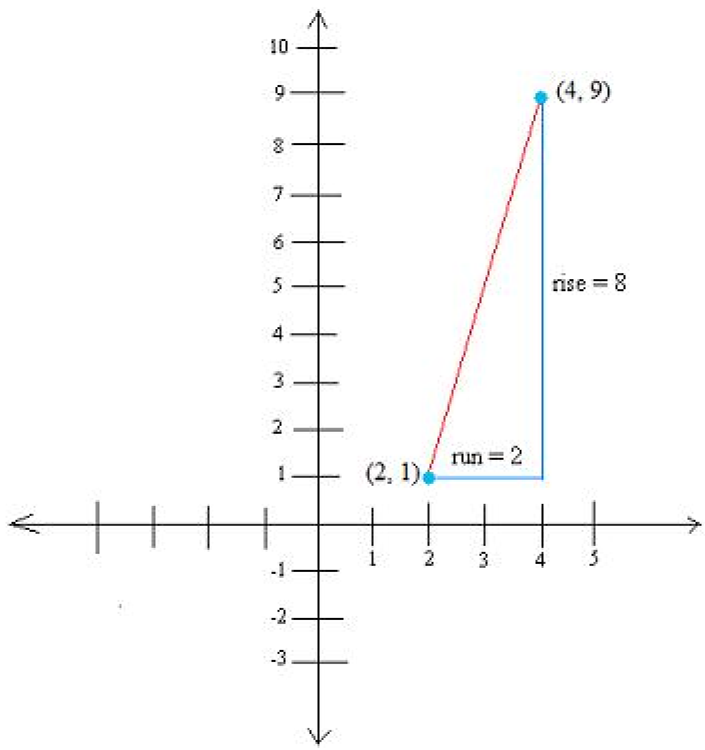
x = 15/4 = 3.75

Coordinate Geometry problems

Slope, midpoint, and distance

Word problem #1

Find the slope, midpoint, and distance using the graph below:



Method # 1

$$m = \frac{\text{rise}}{\text{run}} = \frac{8}{2} = 4$$

Method # 2

$$m = \frac{9-1}{4-2} = \frac{6}{2} = 3$$

The midpoint formula is:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{2 + 4}{2}, \frac{1 + 9}{2}\right) = \left(\frac{6}{2}, \frac{10}{2}\right) = (3, 5)$$

The distance formula is:

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$d = \sqrt{(4 - 2)^2 + (9 - 1)^2} = \sqrt{(2)^2 + (8)^2} = \sqrt{(4 + 64)} = \sqrt{68} = 8.24$$

You get the same answer if you do.

$$d = \sqrt{(2 - 4)^2 + (1 - 9)^2} = \sqrt{(-2)^2 + (-8)^2} = \sqrt{(4 + 64)} = \sqrt{68} = 8.24$$

Linear equations

Word problem #2

A line has a slope of 2 and y-intercept equal to 5, write the slope intercept form
m = 2 and b = 5.
The equation is y = mx + b = 2x + 5

Word problem #3

A line passing through (1, 6) has a slope of 5. Write the sloe intercept form of the equation

Use the slope-point form.

$y - y_1 = m(x - x_1)$
y - 6 = 5 (x - 1)
y -6 = 5x -5
y = 5x - 5 + 6
y = 5x + 1

Word problem #4
]
A line has x-intercept 4 and y-intercept -2. Find the slope intercept form of the line

$$\frac{y}{a} + \frac{x}{b} = 1$$

a = -2 and b = 4

$$\frac{y}{-2} + \frac{x}{4} = 1$$

Get the same denominator

$$\frac{y}{-2} \times \frac{-2}{-2} + \frac{x}{4} = 1$$

$$\frac{-2y}{4} + \frac{x}{4} = 1$$

$$\frac{-2y + x}{4} = 1$$

Multiply both sides by 4.

$$\cancel{4} \times \frac{-2y + x}{\cancel{4}} = 1 \times 4$$

-2y + x = 4

-2y = -x + 4

$$y = \frac{-x}{-2} + \frac{4}{-2}$$

$$y = \frac{x}{2} + -2$$

Word problem #5

Find the slope-intercept form of a line passing through points (2, 3) and (4, 9) This time both m and b are missing, so the first thing to do is to get m and then use m and a point either (2, 3) or (4, 9) to get b

$x_1 = 4$, $y_1 = 9$ and $x_2 = 2$, $y_2 = 3$

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

$$\mathrm{m} = \frac{9-3}{4-2} = \frac{6}{2} = 3$$

So far y = 3x + b

Now we can use the value for m and one point to get b

Although you have two points, It does not matter which point you choose.

Since both points are on the line, they will yield similar results

Choosing (2, 3), x = 2 and y = 3

Substituting 2 for x, 3 for y, and 3 for m into the equation y = 3x + b we get:

3 = 3 × 2 + b

3 = 6 + b

Subtract 6 from both sides

$3 - 6 = 6 - 6 + b$

$-3 = 0 + b$

$-3 = b$

Now we have $b = -3$ and $m = 3$, $y = 3x + -3$

Equations of Parallel lines and perpendicular lines

Find the equation of the line parallel and perpendicular to $y = 2x + -4$ and passing though (-1, 5)

The equation parallel to $y = 2x + -4$ as the same slope as $y = 2x + -4$

So, $y = 2x + b$.

Just substitute x for -1 and y for 5 to get b and you are done.

$5 = 2 \times -1 + b$
$5 = -2 + b$
$5 + 2 = b$
$7 = b$

So, $y = 2x + 7$ is the equation parallel to $y = 2x + -4$ and passing through (-1, 5)

The equation perpendicular to $y = 2x + -4$ has slope equal to the negative reciprocal of 2.
In other words, $2 \times$ "slope of equation perpendicular to $2x + -4$" = -1

Since 2 times $-\frac{1}{2} = -1$, slope of equation perpendicular to $2x + -4 = -\frac{1}{2}$

You can see that it is indeed the negative reciprocal of 2!

$$y = -\frac{1}{2}x + b$$

Just substitute -1 for x and 5 for y to get b and you are done.

$$5 = -\frac{1}{2} \times -1 + b$$

$$5 = \frac{1}{2} + b$$

$$5 - \frac{1}{2} = \frac{1}{2} - \frac{1}{2} + b$$

$$5 - \frac{1}{2} = b$$

$$\frac{5}{1} - \frac{1}{2} = b$$

$$\frac{5}{1} \times \frac{2}{2} - \frac{1}{2} = b$$

$$\frac{10}{2} - \frac{1}{2} = b$$

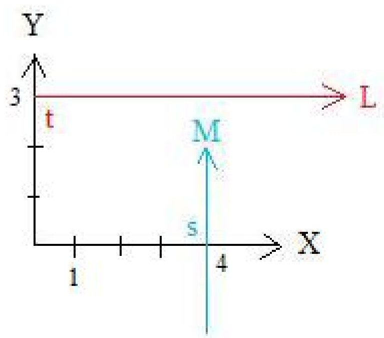
$$\frac{9}{2} = b$$

So, the equation perpendicular to y = 2x + -4 and passing through (-1, 5) is

$$y = -\frac{1}{2}x + \frac{9}{2}$$

Word problem #6

Write the linear equation of line L and line M.



Line L is parallel to x-axis: $y = 3$, x is any number. This makes sense because the general for of a linear equation is $y = mx + b$
Since, the line is parallel to to x-axis, the slope is 0.
So $y = b$ and $b = 3$.
Line M is parallel to y-axis: $x = 4$, y is any number

Right triangle problems

Word problem # 1

The legs of a right triangle are 3 and 4 feet. What is the length of the hypotenuse? Here, you are looking for c or the longest side.

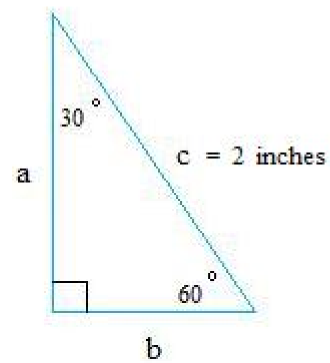
$c^2 = a^2 + b^2$
$c^2 = 3^2 + 4^2$
$c^2 = 9 + 16$
$c^2 = 25$
$c = \sqrt{25}$
$c = 5$

Find the length of the third leg when the length of the hypotenuse is 10 and length of the other leg is 6.
Here, $c = 10$ and either a or b equals 6.

$c^2 = a^2 + b^2$
$10^2 = 6^2 + b^2$
$100 = 36 + b^2$
$100 - 36 = 36 - 36 + b^2$
$64 = b^2$

$\sqrt{64} = b$
$8 \quad = b$

Word problem # 2



The hypotenuse of a $30° - 60° - 90°$ triangle is equal to 2 inches as shown above. Find the other two legs a and b.

$$b = \frac{c}{2}$$

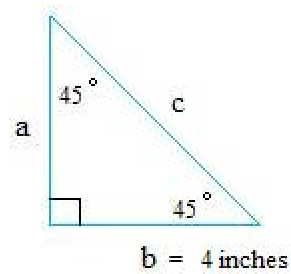
$$b = \frac{2}{2} = 1$$

$$a = b \times \sqrt{3}$$

$$a = 1 \times \sqrt{3}$$

$$a = \sqrt{3} \text{ inches}$$

Word problem # 3



The leg of a $45° - 45° - 90°$ triangle is equal to 4 inches

What is the length of the hypotenuse and the other side?

The other side is also equal to 4 inches.
To get c, use the following formula by replacing a by 4.

$$a = \frac{c \times \sqrt{2}}{2}$$

$$4 = \frac{c \times \sqrt{2}}{2}$$

$$4 \times 2 = \frac{c \times \sqrt{2}}{2} \times 2$$

$$8 = c \times \sqrt{2}$$

$$\frac{8}{\sqrt{2}} = c \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$\frac{8}{\sqrt{2}} = c$$

$$\frac{8 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = c$$

$$\frac{8 \times \sqrt{2}}{\sqrt{4}} = c$$

$$\frac{8 \times \sqrt{2}}{2} = c$$

$$4 \times \sqrt{2} = c$$

Word problem #4

Find the height of an equilateral triangle when a side is equal $\sqrt{3}$ inches.

$$h = \frac{s \times \sqrt{3}}{2}$$

$$h = \frac{\sqrt{3} \times \sqrt{3}}{2} = \frac{\sqrt{3 \times 3}}{2} = \frac{\sqrt{9}}{2} = \frac{3}{2}$$

h = 1.5 inches.

Word problem #5

Find the length of a side of an equilateral whose height is equal to 2 inches.

$$h = \frac{s \times \sqrt{3}}{2}$$

$$2 = \frac{s \times \sqrt{3}}{2}$$

$$2 \times 2 = \frac{s \times \sqrt{3}}{2} \times 2$$

$$4 = \frac{s \times \sqrt{3}}{\cancel{2}} \times \cancel{2}$$

$4 = s \times \sqrt{3}$

$\frac{4}{\sqrt{3}} = s \times \frac{\sqrt{3}}{\sqrt{3}}$

$\frac{4}{\sqrt{3}} = s$

$\frac{4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = s$

$\frac{4 \times \sqrt{3}}{\sqrt{3 \times 3}} = s$

$\frac{4 \times \sqrt{3}}{3} = s$

Word problem #6

The length of a side of a square equals 2 inches. What is the length of the diagonal of the square?

$c = s \times \sqrt{2}$

$c = 2 \times \sqrt{2}$

$c = 2 \times 1.41421$

c = 2.8284 inches.

Word problem #7

Find the length of the diagonal of a square when the length of a side is $\sqrt{2}$

$s = \frac{c \times \sqrt{2}}{2}$

$\sqrt{2} = \frac{c \times \sqrt{2}}{2}$

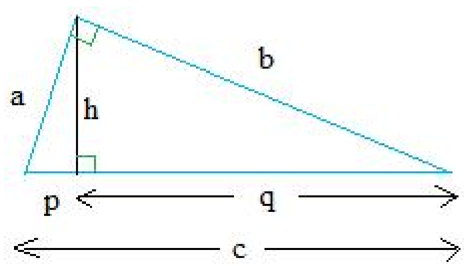
$2 \times \sqrt{2} = \frac{c \times \sqrt{2}}{2} \times 2$

$2 \times \sqrt{2} = c \times \sqrt{2}$

$2 \times \frac{\sqrt{2}}{\sqrt{2}} = c \times \frac{\sqrt{2}}{\sqrt{2}}$

2 = c

Word problem #8



Find the length of a, b, and h if c = 12.5 and p = 2

$a = \sqrt{p \times c} = \sqrt{2 \times 12.5} = \sqrt{25} = 5$

$b = \sqrt{q \times c}$

$q = c - p = 12.5 - 2 = 10.5$

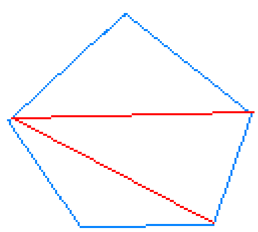
$b = \sqrt{q \times c} = \sqrt{10.5 \times 12.5} = \sqrt{131 \cdot 25} = 11.45$

$h = \sqrt{p \times q} = \sqrt{2 \times 10.5} = \sqrt{21} = 4.58$

Number of diagonals problems

Word problem #1

How many diagonals are there in a pentagon?

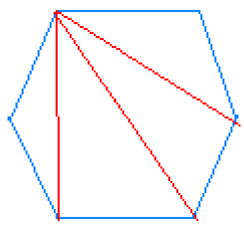


$\frac{n \times d}{2}$ n = 5 and d = 2 (shown in red)

$$\frac{n \times d}{2} = \frac{5 \times 2}{2} = \frac{10}{2} = 5$$

Word problem #2

How many diagonals are there in a hexagon?



$$\frac{n \times d}{2}$$

n = 6 and d = 3 (shown in red)

$$\frac{n \times d}{2} = \frac{6 \times 3}{2} = \frac{18}{2} = 9$$

Pythagorean triples problems

Word problem #1

Find triples when m = 2

$$(2m)^2 + \left(m^2 - 1\right)^2 = \left(m^2 + 1\right)^2$$

$$(2 \times 2)^2 + \left(2^2 - 1\right)^2 = \left(2^2 + 1\right)^2$$

$(4)^2 + (4-1)^2 = (4+1)^2$

$(4)^2 + (3)^2 = (5)^2$ or $25 = 16 + 9$

The triple is (3, 4, 5) This means that the Pythagorean theorem is satisfied.

Word problem #2

Find triples when m = 10

$(2m)^2 + \left(m^2 - 1\right)^2 = \left(m^2 + 1\right)^2$

$(2 \times 10)^2 + \left(10^2 - 1\right)^2 = \left(10^2 + 1\right)^2$

$(20)^2 + (100 - 1)^2 = (100 + 1)^2$

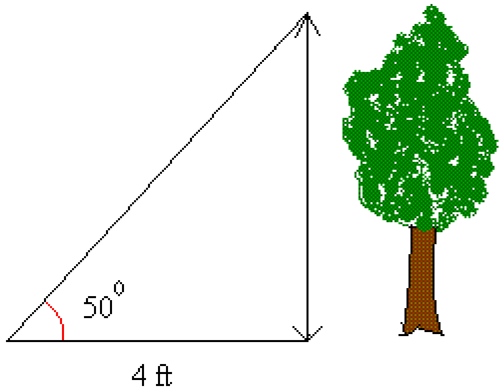
$(20)^2 + (99)^2 = (101)^2$ or $10201 = 400 + 9801$

The triple is (20, 99, 101) This means that the Pythagorean theorem is satisfied.

Basic trigonometric problems

Word problem #1

Find the height of the tree below:



The length of the tree is the side **opposite** to the angle

The known length of 4 feet is **adjacent** to the angle.

So, use the tangent formula since it relates the opposite side and the adjacent side

$$\tan 50° = \frac{\text{opposite}}{4}$$

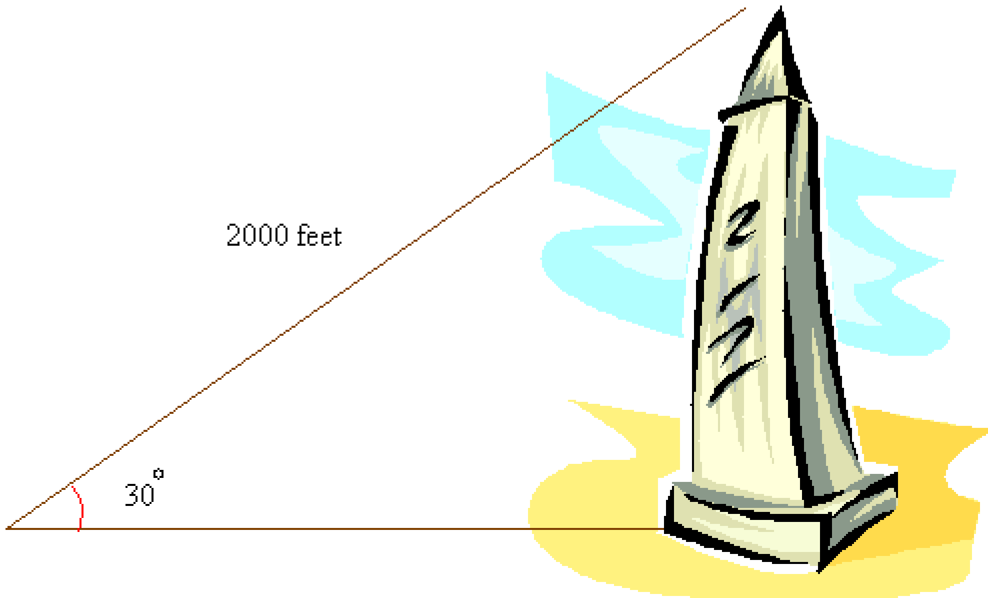
$$1.19 = \frac{\text{opposite}}{4}$$

$$4 \times 1.19 = \cancel{4} \times \frac{\text{opposite}}{\cancel{4}}$$

4.76 feet = opposite

Word problem #2

Find the height of the building below:



The length of the building is the side **opposite** to the angle

The known length of 2000 feet is **hypotenuse**.

So, use the sine formula since it relates the opposite side and the hypotenuse

$$\sin 30° = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \sin 30° = \frac{\text{opposite}}{2000}$$

$$0.5 = \frac{\text{opposite}}{2000}$$

$$2000 \times 0.5 = \cancel{2000} \times \frac{\text{opposite}}{\cancel{2000}}$$

1000 feet = opposite