Illustrated Fractions

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Table of contents

Lesson # 0: Preliminaries

Greatest common factor

The ***greatest common factor*** (GCF) is the largest factor of two numbers. An understanding of ***factor*** is important in order to understand the meaning of GCF.

What are factors?

When two or more numbers are multiplied in a multiplication problem, each number is a factor in the multiplication.

Take a look at the following multiplication problem:

$2 \cdot 8 \cdot 3$.

2 is a factor. 8 is also a factor.

You can find all factors of a number by finding all numbers that divide the number.

Find all factors of 36:

Start with 1. Since 1 divide 36, 1 is a factor.

2 divides 36, so 2 is factor.

3 divides 36, so 3 is a factor.

If you continue with this pattern, you will find that 1, 2, 3, 4, 6, 9, 12, 18, and 36 are all factors of 36.

An easier way to handle the same problem is to do the following:

$1 \cdot 36 = 36$

$2 \cdot 18 = 36$

$3 \cdot 12 = 36$

$4 \cdot 9 = 36$

$6 \cdot 6 = 36$

$9 \cdot 4 = 36$

Note: When the factors start to repeat, you have found them all.

In our example above, the factors started to repeat at $9 \cdot 4 = 36$ because you already had $4 \cdot 9 = 36$.

Therefore, we have found them all.

Now that you have understood how to get the factors of a number, it is going to be straightforward to get the greatest common factor.

Whenever you are talking about greatest common factor, you are referring to 2, 3 numbers, or more. Here, we will concern ourselves with just 2.

The GCF of two numbers is the largest factor of the two numbers.

Examples #1

Find GCF of 16 and 24 written as GCF (16,24).

Using the method described above,

The factors for 16 are 1, 2, 4, **8**, and 16.

The factors for 24 are 1, 2, 3, 4, 6, **8**, 12, and 24.

The largest factor both numbers have in common is 8, so
GCF (16,24) = 8.

Notice that 4 is also a common factor. However, 4 is not the greatest.

Examples #2

Find GCF (7,12)

The factors for 7 are **1** and 7.

The factors for 12 are **1**, 2, 3, 4, 6, and 12

The largest number both factors have in common is 1, so
GCF (7,12) = 1

Note: ***If number a divides b, the greatest common factor is a.***

Examples #3

Find GCF (3,6)

Since 3 divides 6, GCF (3,6) = 3

Resources:

http://www.basic-mathematics.com/factors-calculator.html

A calculator that will list all factors for any number entered.

www.basic-mathematics.com/greatest-common-factor-calculator.html

A calculator that will compute the greatest common factor for you with the click of a button.

www.basic-mathematics.com/greatest-common-factor-game.html

A fun greatest common factor game that will help you test your knowledge of the GCF.

Least common multiple

An understanding of ***multiples*** is important in order to understand the meaning of least common multiple.

The multiples of a number are the answers that you get when you multiply that number by the whole numbers.

Remember that the whole numbers are all numbers from 0 to infinity.

Whole number = {0, 1, 2, 3, 4, 5, 6, 7, 8, ...}

For example, to get the multiples of 4, multiply 4 by 0, 1, 2, 3, 4, 5, ...

I put the dots to show that the set of whole numbers continues forever.

After you perform these multiplications you get:

{0, 4, 8, 12, 16, 20, ...}

The multiples of 9 are all the numbers that you get when you multiply 9 by 0, 1, 2, 3, 4, 5, 6, ...

After you perform these multiplications, you get:

{0, 9, 18, 27, 36, 45, 54, ...}

The ***least common multiple*** (LCM) of two numbers is the smallest number that is a multiple for both numbers.

Example #1: Find LCM of 6 and 9

First list all the multiples of 6

You get {0, 6, 12, **18**, 24, 30, **36**, 42, 48, 54, 60, ……..}

Next, list all the multiples of 9

You get {0, 9, **18**, 27, **36**, 45, 54, 63, 72, 81, 90 ……..}

Carefully examine the two lists and you will see that the smallest number that is a multiple of both 6 and 9 is 18.

You can also write LCM (6, 9) = 6

Of course, 36 is also a common multiple of 6 and 9.However, it is not the smallest.

Example #2: Find LCM of 2,and 3

Multiples of 2 are {0, 2, 4, **6**, 8, 10, ……..}

Multiples of 3 are {0, 3, **6**, 9, 12, 15 ……...}

Again, carefully examine the two lists and you will see that the smallest number that is a multiple of both 2 and 3 is 6.

You can also write LCM (2, 3) = 6

Note: ***When two numbers are prime, the least common multiple is found by simply multiplying the numbers.***

However, what is a prime number?

A ***prime number*** is a number that can be divided **evenly** only by 1 and itself. This means that the number has exactly two factors.

For example, 13 is a prime number because 13 can only be divided evenly by 1 and 13. Any other number we try to divide 13 by will yield a decimal number.

If the number has more than two factors, we say that the number is ***composite***.

For instance, 12 is a composite number because 12 can be divided by numbers other than 1 and 12, such as 2, 3, and 4.

Note that in example #2, since 2 and 3 are prime numbers, we could have just multiplied the numbers to get the least common multiple.

Example #3: Find LCM of 5 and 7

Since 5 and 7 are prime numbers, we can just multiply 5 and 7 to get the least common multiple

$5 \cdot 7 = 35$, so the least common multiple is 35.

Note: ***If number a divides b, the least common multiple is b.***

Example #4: Find LCM of 2 and 4

Since 2 divide 4, LCM (2, 4) = 4

You can always list all multiples of 2.

Multiples of 2 are {2, **4**, 6, 8, 10, 12…….}

Then list all multiples of 4.

Multiples of 4 are {**4**, 8, 12, 16, 20, …….}

However, listing all the multiples for both numbers can take time. If you remember the fact above, it takes a lot less time.

Resources:

www.basic-mathematics.com/least-common-multiple-calculator.html

A calculator that will compute the least common multiple for you with the click of a button.

www.basic-mathematics.com/least-common-multiple-game.html

A fun least common multiple game that will help you test your knowledge of the LCM.

http://www.basic-mathematics.com/prime-number-calculator.html

A calculator that will tell you if a number is prime

Divisibility Rules

Divisibility rules of whole numbers are very useful because they help us quickly determine if a number can be divided by 2, 3, 4, 5, 6, 7, 8 9, and 10 without doing long division.

Divisibility means that you are able to divide a number **evenly**.

For instance, 8 can be divided evenly by 4 because 8/4 = 2. However, 3 cannot divide 8 evenly.

To illustrate the concept, let's say you have a cake and your cake has 8 slices. You can share that cake between you and 3 more people evenly. Each person will get 2 slices.

However, if you are trying to share those 8 slices between you and 2 more people, there is no way you can do this evenly. One person will end up with less cake.

In general, a whole number x divides another whole number y if and only if you can find a whole number n such that

$x \cdot n = y$

For instance, 12 can be divided by 3 because $3 \cdot 4 = 12$

When the numbers are large, use the following divisibility rules:

Rule #1: divisibility by 2

A number is divisible by 2 if its last digit is 0, 2, 4, 6, or 8.

For instance, 8596742 is divisible by 2 because the last digit is 2.

Rule # 2: divisibility by 3:

A number is divisible by 3 if the sum of its digits is divisible by 3.

For instance, 3141 is divisible by 3 because 3+1+ 4+1 = 9 and 9 is divisible by 3.

Rule # 3: divisibility by 4

A number is divisible by 4 if the number represented by its last two digits is divisible by 4.

For instance, 8920 is divisible by 4 because 20 is divisible by 4.

Rule #4: divisibility by 5

A number is divisible by 5 if its last digit is 0 or 5.

For instance, 9564655 is divisible by 5 because the last digit is 5.

Rule # 5: divisibility by 6

A number is divisible by 6 if it is divisible by 2 and 3. Be careful! It is not one or the other. The number must be divisible by both 2 and 3 before you can conclude that it is divisible by 6.

Rule # 6: divisibility by 7

To check divisibility rules for 7, study carefully the following two examples:

Is 348 divisible by 7?

Remove the last digit, which is 8. The number becomes 34.

Then, double 8 to get 16 and subtract 16 from 34.

34 – 16 = 18 and 18 is not divisible by 7.

Therefore, 348 is not divisible by 7.

Is 37961 divisible by 7?

Remove the last digit, which is 1. The number becomes 3796. Then, double 1 to get 2 and subtract 2 from 3796.

3796 – 2 = 3794. Since it is still too big, repeat the process.

Remove the last digit, which is 4. The number becomes 379. Then, double 4 to get 8 and subtract 8 from 379.

379 – 8 = 371. Since it is still too big, repeat the process.

Remove the last digit, which is 1. The number becomes 37. Then, double 1 to get 2 and subtract 2 from 37.

37 – 2 = 35 and 35 is divisible by 7. Therefore, 37961 is divisible by 7.

Rule #7: divisibility by 8

A number is divisible by 8 if the number represented by its last three digits is divisible by 8.

For instance, 587320 is divisible by 8 because 320 is divisible by 8.

Rule #8: divisibility by 9

A number is divisible by 9 if the sum of its digits is divisible by 9.

For instance, 3141 is divisible by 9 because the sum of its digits is divisible by 9.

Rule # 9: divisibility by 10

A number is divisible by 10 if its last digits is 0.

For instance, 522480 is divisible by 10 because the last digit is 0.

Resources:

http://www.basic-mathematics.com/divisibility-test-calculator.html

Use this calculator to determine if a number is divisible by any other number.

http://www.basic-mathematics.com/divisibility-rules-game.html

A game that will help you test how well you know the divisibility rules.

We have now covered all preliminaries. Although the primary goal of this book is to help you visualize fractions, it is also important to have in hand a quick and effective way to solve fraction problems. This is what the GCF, LCM, and divisibility rules will help you do.

We now introduce the concept of fractions next.

Lesson # 1: Introduction to fractions

A ***fraction*** is part of a whole number. Look at the following figures. The shaded portion of each figure is part of the whole figure.

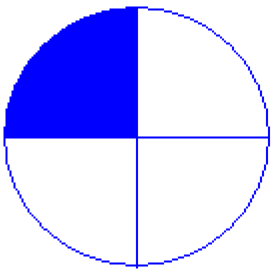


Figure 1

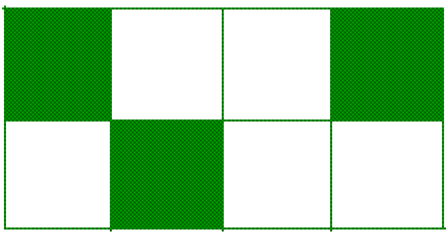


Figure 2

You can also say that the shaded portion is a fraction of the whole figure.

Figure 1 has 4 parts and only 1 part is shaded.

We say that one-fourth of the figure is shaded and

We write $\frac{1}{4}$

We call the number on top, which is 1, the ***numerator***.

We call the number at the bottom, which is 4, the ***denominator***.

We call the line that separates the two numbers ***fraction bar***.

Similarly, **figure 2** has 3 shaded parts out of 8 parts.

We write $\frac{3}{8}$

Once again the number on top, which is 3 is called the numerator and 8 is the denominator.

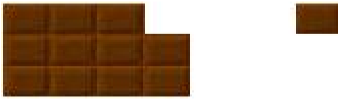
A real life example of fractions is when you buy a pizza and eat only some of that pizza.

Since you did not eat the whole pizza, you only ate a fraction of the pizza.

If your pizza has 10 slices and you eat 4 slices, you can express this as

$$\frac{4}{10}$$

Another good example is a chocolate bar. A chocolate bar is usually made up of pieces a chocolate shaped like rectangles.

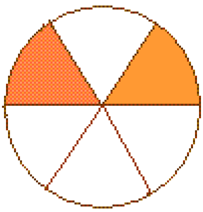


If your chocolate bar has 12 pieces and you eat only 1 piece, you can express this as $\frac{1}{12}$

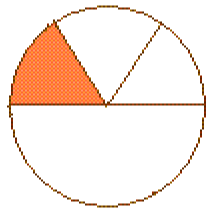
Practice: Introduction to fractions

Write the fractions for each of the following figure:

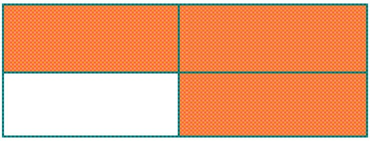
1) __________



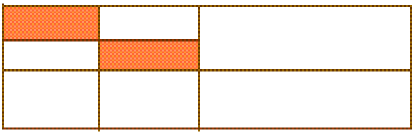
2) __________



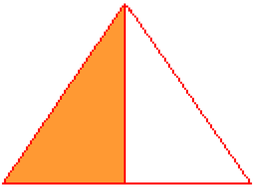
3) __________



4) __________



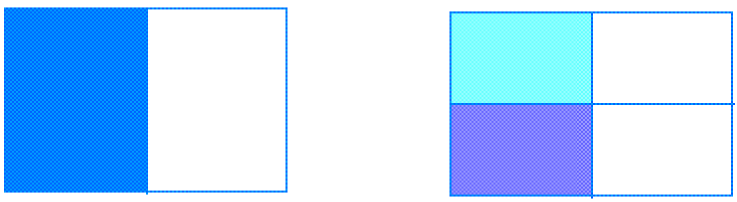
5) __________



Lesson # 2: Equivalent fractions

Look at the following illustration:

Rectangle 1 **Rectangle 2**



$$\frac{1}{2} \quad = \quad \frac{2}{4}$$

Rectangle 1 has 2 parts and only 1 part is shaded, so we can write $\frac{1}{2}$

Rectangle 2 has 4 parts and only 2 parts are shaded, so we can write $\frac{2}{4}$

However, since rectangle 1 is equal to rectangle 2 and the portion that is shaded is the same for both rectangles,

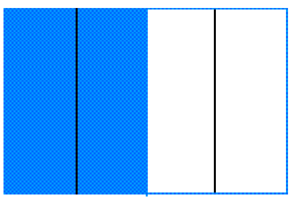
we can conclude that $\frac{1}{2} = \frac{2}{4}$

We call $\frac{1}{2}$ and $\frac{2}{4}$ ***equivalent fractions***. Notice that $\frac{1}{2} = \frac{2}{4} = 0.5$

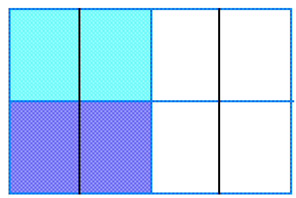
Equivalent fractions are fractions that are equal, but have different numerators and denominators.

Now, what will happen, if in each rectangle, we cut the shaded and the non-shaded area in half? We do this by drawing two lines in each rectangle. The two lines are shown in black

Rectangle 1



Rectangle 2



$$\frac{2}{4} = \frac{4}{8}$$

Rectangle 1 has now 4 parts and only 2 parts are shaded,

so we can write $\frac{2}{4}$

Rectangle 2 has now 8 parts and only 4 parts are shaded,

so we can write $\frac{4}{8}$

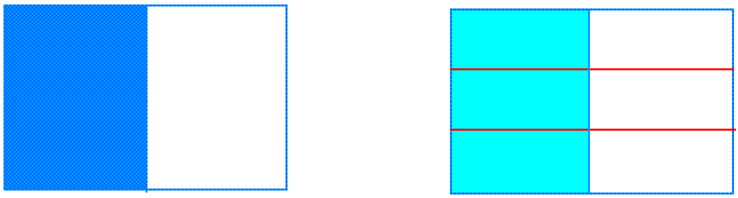
Rectangle 1 is still equal to rectangle 2 and the portion that is shaded is still the same for both rectangles.

We can conclude that $\frac{2}{4} = \frac{4}{8}$

We call $\frac{2}{4}$ and $\frac{4}{8}$ *equivalent fractions*. Notice that $\frac{2}{4} = \frac{4}{8} = 0.5$

You could also break rectangle 2 in 6 parts. Just notice that now 3 parts out of 6 parts are shaded.

Rectangle 1 **Rectangle 2**



$$\frac{1}{2} = \frac{3}{6}$$

Once again, we call $\frac{1}{2}$ and $\frac{3}{6}$ *equivalent fractions*. Notice that $\frac{1}{2} = \frac{3}{6} = 0.5$

Here is what we got so far: $\frac{1}{2} = \frac{2}{4} = 0.5$, $\frac{2}{4} = \frac{4}{8} = 0.5$, and $\frac{1}{2} = \frac{3}{6} = 0.5$

Since all these fractions are equal to 0.5, they are equal to one another.

Putting it all together, we conclude that $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$

Notice that to get from $\frac{1}{2}$ to $\frac{2}{4}$, $\frac{3}{6}$, and $\frac{4}{8}$ we can just multiply both the numerator and the denominator of $\frac{1}{2}$ by 2, 3, and 4.

It is thus not hard to find equivalent fractions for a given fraction. Just multiply the numerator and the denominator of the given fraction by **<u>any</u>** but the **<u>same</u>** natural number greater than 1 such as

2, 3, 4, 5, 6, …….

Example #1

Find two equivalent fractions for the following fractions: $\frac{2}{3}$ and $\frac{4}{5}$

a) $\frac{2}{3}$

$$\frac{2\times 2}{3\times 2}=\frac{4}{6} \text{ and } \frac{2\times 5}{3\times 5}=\frac{10}{15}$$

So, two equivalent fractions for $\frac{2}{3}$ are $\frac{4}{6}$ and $\frac{10}{15}$

b) $\frac{4}{5}$

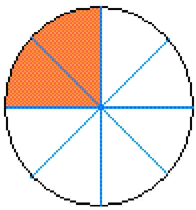
$$\frac{4\times 3}{5\times 3}=\frac{12}{15} \text{ and } \frac{4\times 4}{5\times 4}=\frac{16}{20}$$

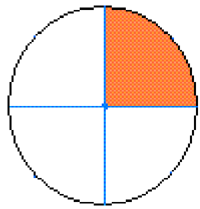
So, two equivalent fractions for $\frac{4}{5}$ are $\frac{12}{15}$ and $\frac{16}{20}$

Practice: **Equivalent fractions**

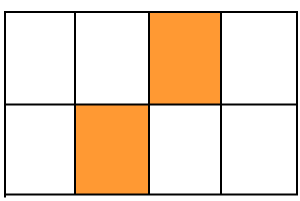
Write the equivalent fractions for each pair of figure

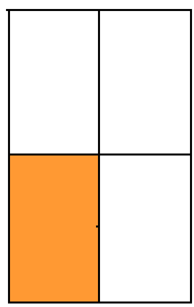
1) ____________ ____________





2) ____________ ____________





Write two equivalent fractions for each number

3) $\frac{5}{6}$ = ______ ______

4) $\frac{4}{10}$ = ______ ______

Note: It is important to know how to get equivalent fractions when adding and subtracting fractions with unlike denominators, which will be covered in lesson #8

Lesson # 3: Simplifying fractions

Simplifying a fraction simply means that you will look for an equivalent fraction that has the smallest numbers you can get for the numerator and the denominator.

Note: This lesson will use the GCF as a shortcut. Make sure you understand how to find the GCF.

Simplifying fraction is then the reverse of finding equivalent fractions. I outlined some steps below before using the GCF:

Step #1

Just divide the numerator and the denominator by 2 until the numerator and the denominator are no longer divisible by 2 or cannot be divided by 2 anymore. If the numerator and the denominator cannot be divided by 2 at the same time, go to **Step #2.**

Step #2

Then, divide the numerator and the denominator by 3 until the numerator and the denominator are no longer divisible by 3 or cannot be divided by 3 anymore. If the numerator and the denominator cannot be divided by 3 at the same time, go to **Step #3.**

Step #3

Divide the numerator and the denominator by 4 until the numerator and the denominator are no longer divisible by 4 or cannot be divided by 4 anymore. If the numerator and the denominator cannot be divided by 4 at the same time, go to **Step #4**

……….

……….

……….

And so forth

As you may have noticed, you just keep dividing by the next higher number. If the next higher number cannot divide the numerator and the denominator evenly, you try the next higher number and so forth…

Example #1

Simplify $\frac{18}{36}$

Step #1: Divide 18 and 36 by 2

$$\frac{18}{36} = \frac{18 \div 2}{36 \div 2} = \frac{9}{18}$$

Since 2 cannot divide 9 and 18 at the same time, go to step #2.

Step #2: Divide 9 and 18 by 3.

Step #2

$$\frac{9}{18} = \frac{9 \div 3}{18 \div 3} = \frac{3}{6}$$

Since 3 can divide 3 and 6 at the same time, repeat step #2.

$\frac{3}{6} = \frac{3 \div 3}{6 \div 3} = \frac{1}{2}$ and you are done since 1 and 2 cannot be divided anymore evenly by any number.

Example #2

1) Simplify $\frac{84}{126}$

Step #1: Divide 84 and 126 by 2

Since 2 cannot divide 42 and 63 at the same time, go to step #2.

Step #2: Divide 42 and 63 by 3

$$\frac{42}{63} = \frac{42 \div 3}{63 \div 3} = \frac{14}{21}$$

Since 3 cannot divide 14 and 21 at the same time, go to step #3.

Step #3: Divide 14 and 21 by 4

Since 4 cannot divide 14 and 21 at the same time, go to step #4.

Step #4: Divide 14 and 21 by 5

Since 5 cannot divide 14 and 21 at the same time, go to step #5.

Step #5: Divide 14 and 21 by 6

Since 6 cannot divide 14 and 21 at the same time, go to step #6.

Step #6: Divide 14 and 21 by 7

$\frac{14}{21} = \frac{14 \div 7}{21 \div 7} = \frac{2}{3}$ and you are done since 2 and 3 cannot be divided

anymore evenly by any number.

The method outlined above works very well. However, it can quickly become time consuming, especially if you are working with big number. This is where the greatest common factor comes into place.

In example #1, simplifying $\frac{18}{36}$ gave $\frac{1}{2}$

We get $\frac{1}{2}$ by dividing 18 and 36 by 18 and 18 is the greatest common

factor for 18 and 36.

We already explained in lesson #0 (greatest common factor) how to find the greatest common factor. We will briefly review it here.

Find greatest common factor for 18 and 36 written as GCF (18, 36)

First, list all factors of 18: 1, 2, 3, 6, 9, and **18**

Second, list all factors of 36: 1, 2, 3, 4, 6, 9, 12, **18**, and 36

GCF (18, 36) = 18 because 18 is the greatest factor 18 and 36 have in Common.

Note that 2 and 9 are also common factors, but they are not the greatest.

Notice that 18 divides 36, so as we said before, so we can immediately conclude that 18 is the greatest common Factor.

In example #2, simplifying $\frac{84}{126}$ gave $\frac{2}{3}$

We get $\frac{2}{3}$ by dividing 84 and 126 by 42 and 42 is the greatest common

factor for 84 and 126

Find greatest common factor for 84 and 126 written as GCF (84, 126)

First, list all factors of 84: 1, 2, 3, 4, 6, 7, 12, 14, 21, 28**,** **42**, and 84

Second, list all factors of 126: 1, 2, 3, 6, 7, 14, 18, 21, **42**, 63 and 126

GCF (84, 126) = 42 because 42 is the greatest factor 84 and 126 have in common.

Examples #3

Simplify $\frac{15}{25}$

Find GCF (15, 25)

Factors of 15 are 1, 3, **5**, and 15

Factors of 25 are 1, **5**, and 25

GCF (15, 25) = 5 because 5 is the greatest factor 15 and 25 have in common.

Just divide 15 and 25 by GCF (15, 25). We already know the GCF. It is 5. We just explained this above.

So, $\frac{15 \div 5}{25 \div 5} = \frac{3}{5}$

Examples #4

Simplify $\frac{12}{42}$

First, find GCF (12, 42)

Factors of 12 are 1, 2, 3, 4, **6**, and 12

Factors of 42 are 1, 2, 3, **6**, 7, 21, and 42

GCF (12, 42) = 6

Just divide both 12 and 42 by 6

We get:

$$\frac{12 \div 6}{42 \div 6} = \frac{2}{7}$$

Resources:

http://www.basic-mathematics.com/reduce-fractions-calculator.html

A calculator that will simplify any fraction.

Simplifying fractions is useful when we try to make an answer look simpler or make the numerator and the denominator look small. Sometimes, simplifying fractions before adding, subtracting, dividing, and/or multiplying can make calculation less complicated and confusing.

Practice: **simplifying fractions**

Exercises: Simplify the following fractions

1) $\frac{2}{6}$ ________

2) $\frac{6}{12}$ ________

3) $\frac{3}{9}$ ________

4) $\frac{4}{8}$ ________

5) $\frac{3}{12}$ ________

6) $\frac{6}{18}$ ________

7) $\frac{7}{21}$ ________

8) $\frac{2}{16}$ ________

Lesson # 4: Proper, improper fractions and mixed numbers

A ***proper fraction*** is a fraction that has a smaller numerator than a denominator. The following are all proper fractions:

$\frac{1}{4}$ $\frac{5}{11}$ $\frac{4}{10}$ $\frac{3}{5}$ $\frac{50}{60}$

An ***improper fraction*** is a fraction that has a bigger numerator than a denominator. The following are all improper fractions:

$\frac{6}{5}$ $\frac{7}{2}$ $\frac{2}{1}$ $\frac{5}{3}$ $\frac{11}{5}$

A ***mixed number*** is a number that is a combination of a whole number and a proper fraction. The following are mixed numbers.

$2\frac{1}{4}$ $1\frac{2}{5}$ $5\frac{1}{8}$ $9\frac{5}{8}$

Notice that $2\frac{1}{4}$ means 2 and $\frac{1}{4}$ or $2 + \frac{1}{4}$

Therefore, $2\frac{1}{4}$ is per se a simplified format for $2 + \frac{1}{4}$

Nonetheless, know this once and for all that there is an addition sign between the whole number and the proper fraction.

Mixed numbers can be converted to improper fractions and vice versa.

More on this is coming in the next lesson.

When adding and subtracting fractions, it is not necessary to convert mixed numbers to improper fractions before adding or subtracting. Depending on the problem, one or the other could be used.

However, when multiplying and dividing fractions, it is **imperative** to convert all mixed number(s) to improper fractions before multiplying or dividing.

The only exception is when you are using rectangles or circles to perform the multiplication. We show this in lesson 9.

Lesson # 5: Renaming mixed numbers and improper fractions

Sometimes, you may need to rename or write an improper fraction as a mixed number.

Here is a simple scenario: You have a cake to share between 3 people and your cake has 10 slices.

Question: How many slices can each person get? To answer this

question, you will need to convert $\frac{10}{3}$ into a mixed number.

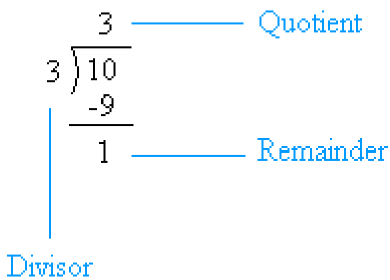
You can argue that each person can get **3** complete slices since $3 \times 3 = 9$

Then, there is 1 slice to share between 3 people or $\mathbf{\frac{1}{3}}$

Combining 3 and $\mathbf{\frac{1}{3}}$**,** we get the mixed number $3\frac{1}{3}$

You can get the same answer when doing long division. In this case, the quotient is the whole number part and the remainder over the divisor is the proper fraction. This is illustrated the next.

Long division gives:



Quotient = 3 and $\dfrac{remainder}{divisor} = \dfrac{1}{3}$ so, we get the same answer $3\,\dfrac{1}{3}$

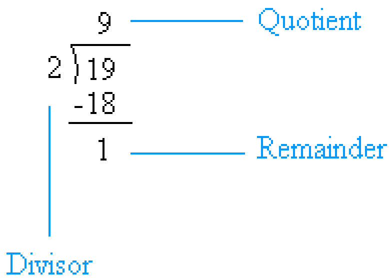
In general, use the following formula to convert improper fractions to mixed numbers.

$$Quotient\,\frac{remainder}{divisor}$$

Examples Convert $\frac{19}{2}$ and $\frac{18}{5}$ into mixed numbers.

a) $\frac{19}{2}$

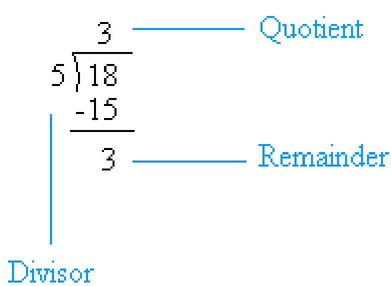
Long division gives:



So $\frac{19}{2} = quotient \frac{reminder}{divisor} = 9\frac{1}{2}$

b) $\frac{18}{5}$

Long division gives:



So, $\frac{18}{5} = 3\frac{3}{5}$

Renaming mixed numbers to improper fractions is a lot easier. Just multiply the whole number part by the denominator.

Then, add the result to the numerator. Keep the same denominator.

Examples Convert $3\frac{3}{5}$ and $9\frac{1}{2}$ into improper fractions

a) $3\frac{3}{6} = \frac{3 \times 6 + 3}{6} = \frac{18 + 3}{6} = \frac{21}{6}$

b) $9\frac{1}{2} = \frac{9 \times 2 + 1}{2} = \frac{18 + 1}{2} = \frac{19}{2}$

c) $4\frac{3}{5} = \frac{4 \times 5 + 3}{5} = \frac{20 + 3}{5} = \frac{23}{5}$

Practice: **proper, improper fractions, and mixed numbers**

Exercise A: Find the mixed number for each of the following improper fraction.

1) $\frac{19}{5}$ ________ 2) $\frac{32}{6}$ ________

3) $\frac{22}{4}$ ________ 4) $\frac{15}{2}$ ________

5) $\frac{10}{3}$ ________ 6) $\frac{9}{5}$ ________

Exercise B: Rename these mixed numbers as improper fractions

1) $2\frac{1}{5}$ ________ 2) $3\frac{2}{4}$ ________

3) $1\frac{2}{3}$ ________ 4) $4\frac{5}{2}$ ________

5) $10\frac{1}{3}$ ________ 6) $3\frac{2}{5}$ ________

Lesson # 6: Comparing fractions

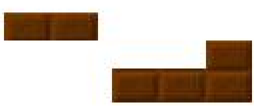
We compare fractions using greater than (>) and less than sign (<).

For example, $\frac{2}{6} < \frac{3}{6}$ or $\frac{3}{6} > \frac{2}{6}$

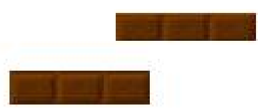
How do we know $\frac{2}{6} < \frac{3}{6}$?

Take a look at the following illustrations with chocolate bars.

$$\frac{6}{6} = 1$$



$\frac{2}{6}$



$\frac{3}{6}$

Eating 2 bars out of 6 bars (Represented as $\frac{2}{6}$) is less than eating 3 bars out of 6 bars (Represented as $\frac{3}{6}$)

So, we say that $\frac{2}{6} < \frac{3}{6}$

Notice that the denominator is the same for both fractions. We draw from the above the following conclusion:

When two fractions have the same denominator, the smaller fraction is the one with the smaller numerator.

Example #1

Compare $\frac{4}{15}$ and $\frac{3}{15}$

$\frac{4}{15}$ is bigger than $\frac{3}{15}$ because 4 is bigger than 3. Thus, $\frac{4}{15} > \frac{3}{15}$

You can also write $\frac{3}{15} < \frac{4}{15}$ which means that $\frac{3}{15}$ is smaller than $\frac{4}{15}$

And again the reason $\frac{3}{15}$ is smaller than $\frac{4}{15}$ is because it has a

smaller numerator

Example #2

Compare the following fractions and put them in order from least to greatest.

$\frac{3}{7}$ $\quad$ $\frac{1}{7}$ $\quad$ $\frac{2}{7}$

$1 < 2$ and $2 < 3$, so the right order is $\frac{1}{7} < \frac{2}{7} < \frac{3}{7}$

Now, what do we do when the fractions have different denominators?

For example, given $\frac{2}{3}$ and $\frac{3}{4}$, how do we know which one is greater?

The trick is to look for equivalent fractions for both $\frac{2}{3}$ and $\frac{3}{4}$ that have the same denominators.

To get the same denominator, find LCM (3,4).

At this point, it is expected that you already know how to get the least common multiple. If you are still struggling, follow the steps outlined in lesson #0 and use the calculator listed as a resource to check your answer.

Anyway, LCM (3,4) = 12.

Since LCM (3,4) = 12, 12 is the common denominator.

Notice that now, you should multiply the denominator of $\frac{2}{3}$ that is 3 by 4 and the denominator of $\frac{3}{4}$ that is 4 by 3 to get the same denominator.

Just remember that you have to multiply your numerator by the same number you multiplied the denominator.

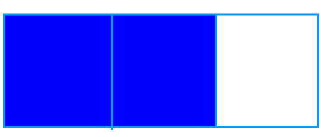
You need to do $\frac{2 \times 4}{3 \times 4}$ and $\frac{3 \times 3}{4 \times 3}$

And you will get equivalent fractions with the same denominators

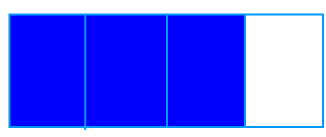
$\frac{2 \times 4}{3 \times 4} = \frac{8}{12}$ and $\frac{3 \times 3}{4 \times 3} = \frac{9}{12}$

Since $\frac{8}{12} < \frac{9}{12}$ then $\frac{2}{3} < \frac{3}{4}$

Here is an illustration for the situation above

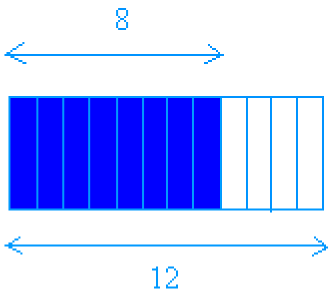


$\frac{2}{3}$

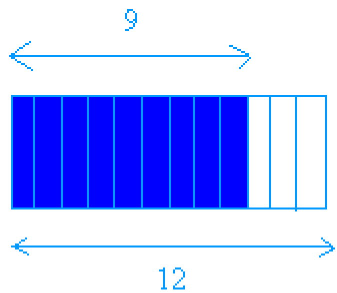


$\frac{3}{4}$

Equivalent fraction for $\frac{2}{3}$



Equivalent fraction for $\frac{3}{4}$



$$\frac{8}{12} < \frac{9}{12}$$

Once again, since $\frac{8}{12}$ is smaller than $\frac{9}{12}$ then $\frac{2}{3}$ is smaller than $\frac{3}{4}$

A good understanding of how to get equivalent is important when adding or subtracting fractions with unlike denominators. Make sure you understand this lesson very well. We will practice with a few more examples to make sure you understand.

Example #1

Compare $\frac{1}{2}$ and $\frac{1}{3}$

Since LCM (2,3) = 6, equivalent fractions are

$\frac{1 \times 3}{2 \times 3} = \frac{3}{6}$ and $\frac{1 \times 2}{3 \times 2} = \frac{2}{6}$

$\frac{3}{6} > \frac{2}{6}$ so $\frac{1}{2} > \frac{1}{3}$

There is a quicker way to compare $\frac{1}{2}$ and $\frac{1}{3}$

$\frac{1}{2}$ and $\frac{1}{3}$ have the same numerator and it is 1.

$\frac{1}{2}$ means to break 1 into 2 equal pieces

$\frac{1}{3}$ means to break 1 into 3 equal pieces

Let's say 1 is a dollar, which share will you choose?

I am pretty sure you will choose to break a dollar into 2 equal pieces because your share will be bigger than breaking a dollar into 3 pieces. In general, when the fractions have the same numerator, the smaller fraction is the one with the bigger denominator

Example #2

Compare $\frac{3}{5}\ and\ \frac{2}{3}$

Since LCM (5,3) = 15, equivalent fractions are

$$\frac{3 \times 3}{5 \times 3} = \frac{9}{15} \quad \text{and} \quad \frac{2 \times 5}{3 \times 5} = \frac{10}{15}$$

$$\frac{9}{15} < \frac{10}{15} \quad \text{so} \quad \frac{3}{5} < \frac{2}{3}$$

Example #3

Compare $\frac{5}{17}$ and $\frac{1}{3}$

Since LCM (17, 3) = 51, equivalent fractions are

$$\frac{5 \times 3}{17 \times 3} = \frac{15}{51} \quad \text{and} \quad \frac{1 \times 17}{3 \times 17} = \frac{17}{51}$$

$$\frac{15}{51} < \frac{17}{51} \quad \text{so} \quad \frac{5}{17} < \frac{1}{3}$$

Useful tips to keep in mind when comparing fractions.

Tip #1: When two fractions have the same denominators, the smaller fraction is the one with the smaller numerator**.**

Tip #2: When the fractions have the same numerators, the smaller fraction is the one with the bigger denominator

Resources:

www.basic-mathematics.com/compare-fractions-calculator.html

A calculator that will compare two fractions and tell you which one is the bigger.

http://www.basic-mathematics.com/comparing-fractions-game.html

A comparing fractions game to see how well you can compare fractions.

Practice: **Comparing fractions**

Exercises: Compare the following fractions. Tell whether the first fraction is smaller (<) or greater (>) than the second fraction. Replace the question mark with your answer

1) $\frac{2}{3}$? $\frac{3}{4}$

2) $\frac{3}{4}$? $\frac{1}{2}$

3) $\frac{3}{7}$? $\frac{4}{6}$

4) $\frac{2}{5}$? $\frac{3}{6}$

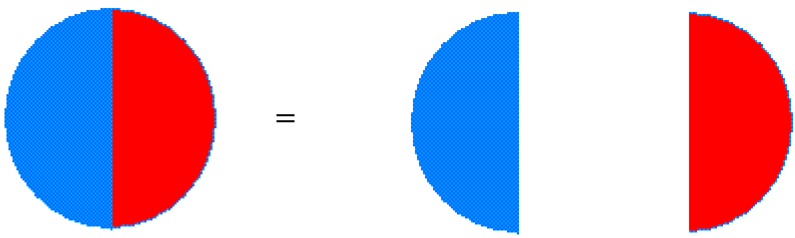
5) $\frac{3}{5}$? $\frac{3}{4}$

6) $\frac{6}{7}$? $\frac{7}{8}$

Lesson # 8: Adding fractions

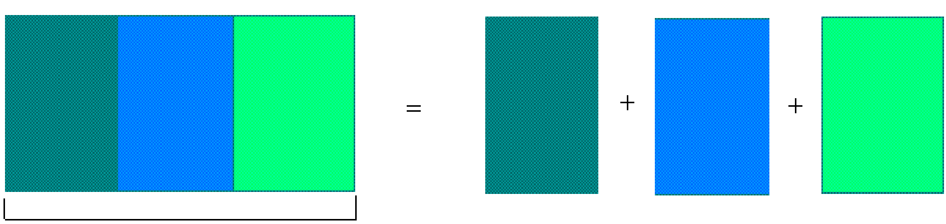
Our goal in this lesson is to illustrate and then show a quick way to get the same answer. Take a close look at the following two figures.

Figure 1



$$1 = \frac{1}{2} + \frac{1}{2}$$

Figure 2



$$1 = \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$$

In **figure 1,** the circle is broken down into 2 parts

So each part is half or $\frac{1}{2}$

Notice that two-halves is equal to one

You could get the same answer if you do $\frac{1}{2}+\frac{1}{2}=\frac{1+1}{2}=\frac{2}{2}=1$

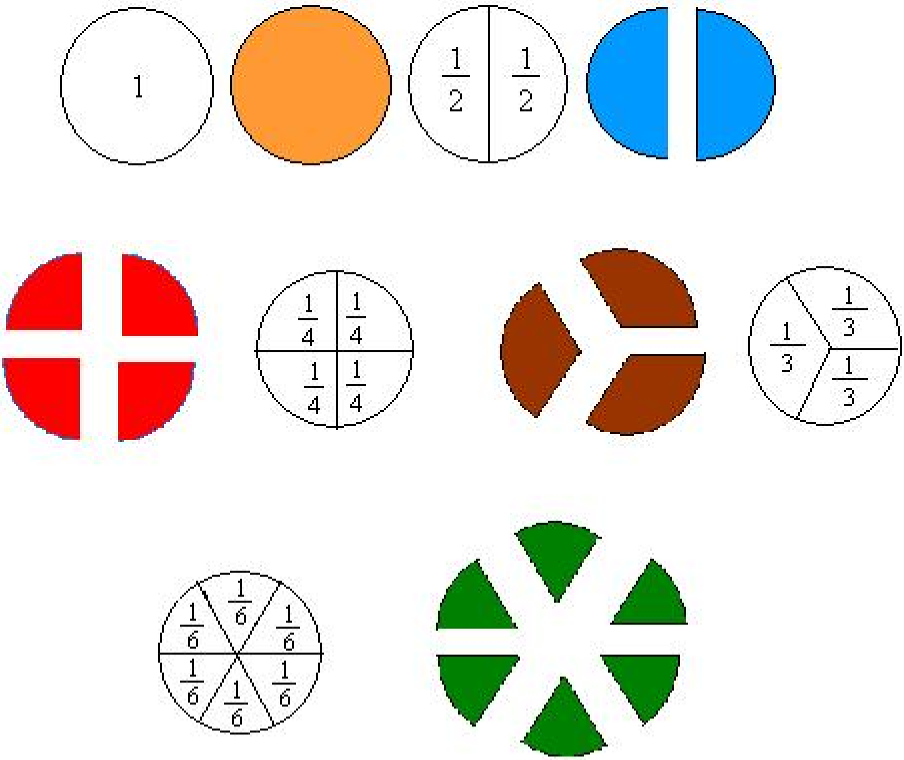
Similarly, three-thirds is equal to one and adding the three parts in **figure 2** gives:

$$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=\frac{1+1+1}{3}=\frac{3}{3}=1$$

Let us explore this concept further with modeling. Study the models given below carefully!

We will call these parts of a circle ***pattern blocks***. Notice that we have not modeled $\frac{1}{5}, \frac{1}{7}, \frac{1}{8}$ and so forth.

The goal is not to model every fraction, but to give you a foundation you can build upon.



Here is the model:

Orange = 1

Blue = $\frac{1}{2}$ = half

Red = $\frac{1}{4}$ = one- fourth

Brown = $\frac{1}{3}$ = one-third

Green = $\frac{1}{6}$ = one-sixth

Now, let's play with these pattern blocks by doing addition.

Quick important facts:

Putting two red blocks next to each other is equal to
one-fourth + one-fourth = two-fourths

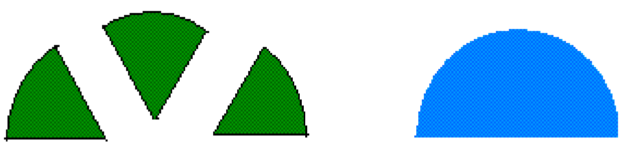
Putting two brown blocks next to each other is equal to
one-third + one-third = two-thirds

Putting 5 green blocks next to each other is equal to:
one-sixth + one-sixth + one-sixth + one-sixth + one-sixth = five-sixths

Putting 6 green blocks next to each other is equal to six-sixths = one

Examples #1 : $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}$

$\frac{1}{6}+\frac{1}{6}+\frac{1}{6}$ **=**



$$\frac{1}{6} \quad + \quad \frac{1}{6} \quad + \quad \frac{1}{6} \qquad = \qquad \frac{1}{2}$$

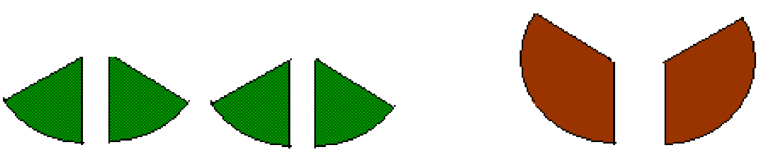
Notice that putting 3 green blocks next to one another is equal 1 big blue block or half.

You can get the same answer if you do

$$\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{1+1+1}{6}=\frac{3}{6}=\frac{3\div 3}{6\div 3}=\frac{1}{2}$$

Examples #2 : $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}$

As shown in example #1, to get the answer for $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}$, put 4 green blocks next to each other



$$\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

Notice that putting 2 green blocks next to each other is equal 1 big brown block and 2 big brown blocks is equal two-thirds.

You could get the same answer if you do

$$\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{1+1+1+1}{6}=\frac{4}{6}=\frac{4\div 2}{6\div 2}=\frac{2}{3}$$

From these examples, we draw the following rules:

We can only add blocks with the same size. This will result in fractions having the same denominator.

Fractions can be added only if the denominators are the same

When adding fractions that have the same denominators, just add the numerators.

We don't add denominators.

Other examples:

1) $\frac{1}{4}+\frac{1}{4}=\frac{2}{4}$

2) $\frac{1}{5}+\frac{2}{5}=\frac{3}{5}$

3) $\frac{6}{7}+\frac{4}{7}=\frac{10}{7}$

4) $\frac{2}{12}+\frac{3}{12}+\frac{1}{12}+\frac{4}{12}=\frac{10}{12}$

5) $\frac{1}{8}+\frac{2}{8}+\frac{1}{8}+\frac{1}{8}+\frac{2}{8}=\frac{7}{8}$

Resources:

www.basic-mathematics.com/adding-fractions-calculator.html

A calculator that will add two fractions and display the answer.

http://www.basic-mathematics.com/adding-fractions-game.html

See how well you can add fractions with this game.

Practice: adding fractions with the same denominator

1) $\frac{1}{2}+\frac{3}{2}=$

2) $\frac{2}{3}+\frac{5}{3}=$

3) $\frac{3}{4}+\frac{6}{4}=$

4) $\frac{4}{2}+\frac{1}{2}=$

5) $\frac{2}{5}+\frac{3}{5}=$

6) $\frac{8}{4}+\frac{4}{4}=$

7) $\frac{1}{8}+\frac{1}{8}=$

8) $\frac{6}{12}+\frac{8}{12}=$

9) $\frac{5}{10}+\frac{4}{10}=$

10) $\frac{8}{9}+\frac{1}{9}=$

Adding fractions with unlike denominators

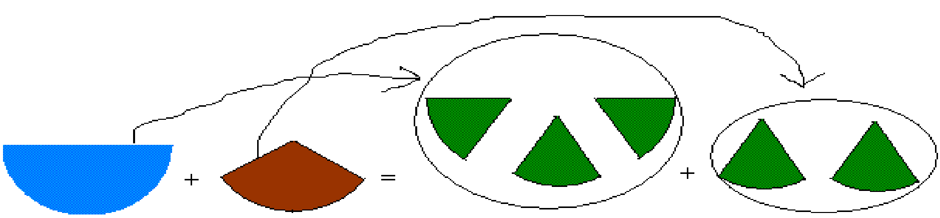
Now, how do we add fractions with different denominators? The following rules already mentioned above are so important, I shall emphasize them again.

Rule #1: ***Fractions can be added only if the denominators are the same***

Rule #2: ***When adding fractions that have the same denominators, just add the numerators.***

Let add $\frac{1}{2}$ and $\frac{1}{3}$

First, let us add with pattern blocks



$$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

Notice that the blue block is traded or swapped with 3 green blocks and the brown block is traded or swapped with 2 green blocks.

You could get the answer if you use the least common multiple. This is done next!

According to rule #1, the denominators should be the same before adding, so we need to look for equivalent fractions that have the same denominators.

The process of looking for equivalent fractions was already demonstrated in the lessons about comparing Fractions.

We shall repeat them here for the sake of making this crystal clear.

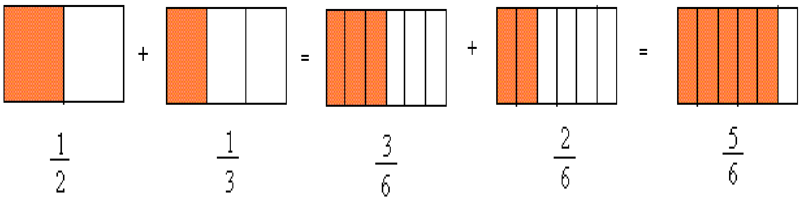
Since LCM (2,3) = 6, equivalent fractions are

$\frac{1 \times 3}{2 \times 3} = \frac{3}{6}$ and $\frac{1 \times 2}{3 \times 2} = \frac{2}{6}$

According to rule #2, you can just add the numerators since both fractions have the same denominators

So, $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{3+2}{6} = \frac{5}{6}$

Here is another illustration for adding $\frac{1}{2}$ and $\frac{1}{3}$



Example #1

$$\frac{2}{5}+\frac{2}{3}$$

Since LCM (5, 3) = 15, equivalent fractions are

$$\frac{3\times 3}{5\times 3}=\frac{9}{15} \quad \text{and} \quad \frac{2\times 5}{3\times 5}=\frac{10}{15}$$

So, $\frac{3}{5}+\frac{2}{3}=\frac{9}{15}+\frac{10}{15}=\frac{9+10}{15}=\frac{19}{15}$

Example #2

$$\frac{5}{17}+\frac{1}{3}$$

Notice that 17 and 3 are prime, so the LCM is $17 \cdot 3 = 51$

Since LCM (17, 3) = 51, equivalent fractions are

$$\frac{5 \times 3}{17 \times 3} = \frac{15}{51} \quad \text{and} \quad \frac{1 \times 17}{3 \times 17} = \frac{17}{51}$$

So, $\frac{5}{17} + \frac{1}{3} = \frac{15}{51} + \frac{17}{51} = \frac{15 + 17}{51} = \frac{32}{51}$

Example #3

$$\frac{3}{2} + \frac{5}{4}$$

We saw in the lesson about LCM that ***if number a divides b, the least common multiple is b***

Since 2 divides 4, LCM (2,4) = 4. Thus the common denominator is 4

Just multiply 2 by 2 to get the same denominator. There is no need to multiply 4 by any number since it is already 4.

Equivalent fractions for $\frac{3}{2}$ is $\frac{3 \times 2}{2 \times 2} = \frac{6}{4}$

So, $\frac{3}{2} + \frac{5}{4} = \frac{6}{4} + \frac{5}{4} = \frac{6 + 5}{4} = \frac{11}{4}$

Practice: **Adding fractions with different denominators**

1) $\frac{2}{3}+\frac{3}{4}=$

2) $\frac{1}{2}+\frac{2}{5}=$

3) $\frac{3}{4}+\frac{4}{2}=$

4) $\frac{4}{6}+\frac{1}{2}=$

5) $\frac{8}{5}+\frac{1}{3}=$

6) $\frac{4}{14}+\frac{2}{7}=$

7) $\frac{2}{8}+\frac{1}{6}=$

8) $\frac{8}{2}+\frac{4}{10}=$

9) $\frac{1}{7}+\frac{4}{5}=$

10) $\frac{8}{2}+\frac{3}{9}=$

Renaming mixed numbers and improper fractions: Revisited with patterns blocks

Converting an improper fraction into a mixed number is the process of trading as many parts as you can into wholes.

In order words, you will try to make as many 1s as you can with the parts.

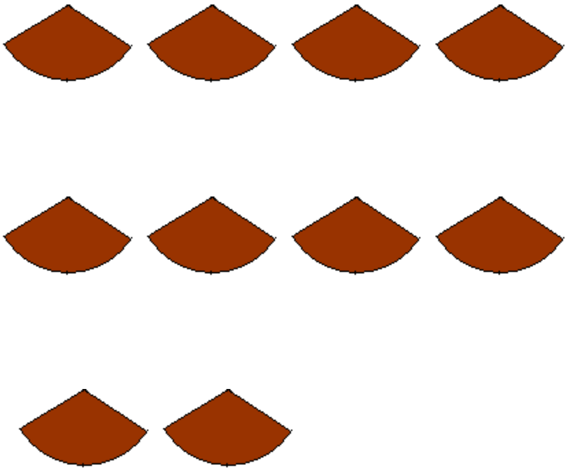
$\frac{10}{3}$ and $\frac{19}{2}$ were already converted into mixed numbers using long division. Let's do it now with pattern blocks.

The trick is to look at the denominator of the fraction so you know which pattern block to use.

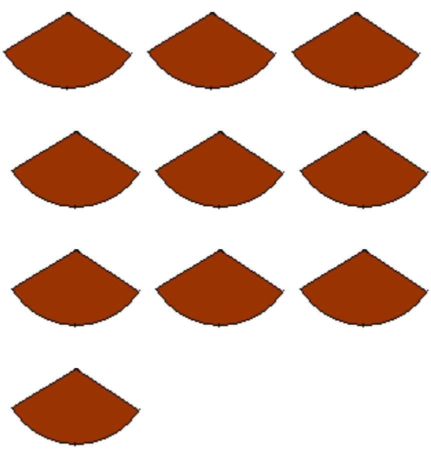
For example, the denominator of $\frac{10}{3}$ is 3.

Therefore, you will try to make as many 1s as possible with the brown block or one-third

Then, since the numerator is 10, you need to lay down 10 brown blocks.



Then, group them into piles of 1s



Notice that three brown blocks make 1. Since there are 3 groups of 3 brown blocks, it is equal to 3.

The leftover is $\frac{1}{3}$

Your mixed number is $3\frac{1}{3}$

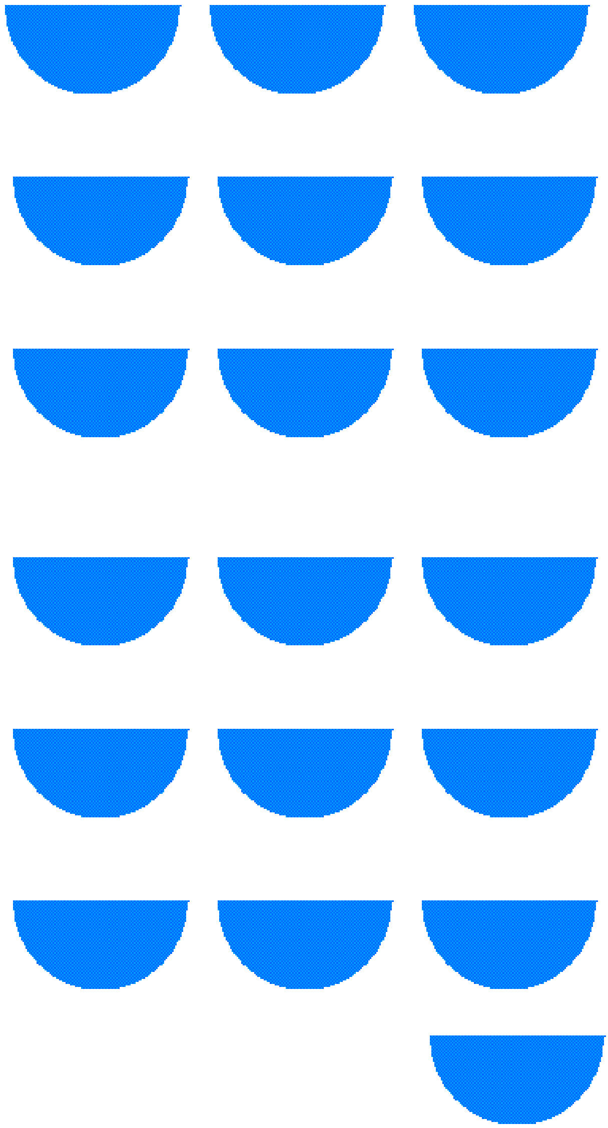
We will now do the same thing for $\frac{19}{2}$

The denominator of $\frac{19}{2}$ is 2.

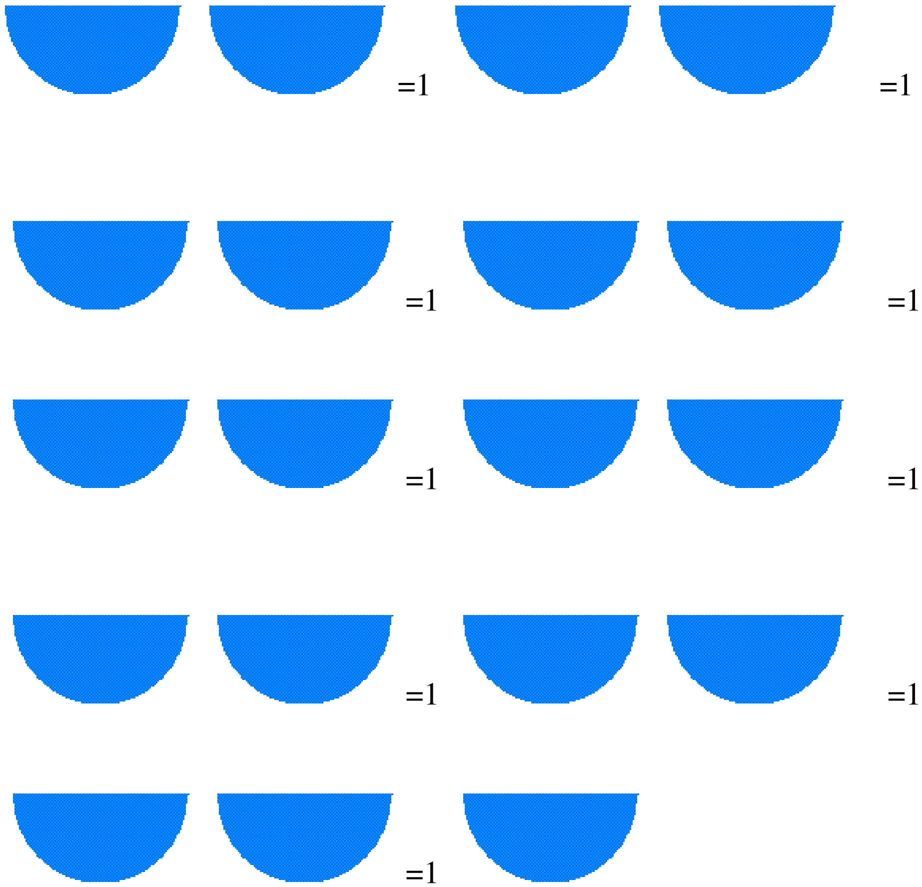
Therefore, you will try to make as many 1s as possible with the blue block or half

Then, since the numerator is 19, you need to lay down 19 blue blocks.

This is shown on the next page.



Then, group them into piles of 1s



Two blue blocks make 1. Since there are 9 groups of 2 blue blocks, it is equal to 9.

The leftover is $\frac{1}{2}$

Your mixed number is $9\ \frac{1}{2}$

Converting a mixed number into an improper fraction is the reverse process.

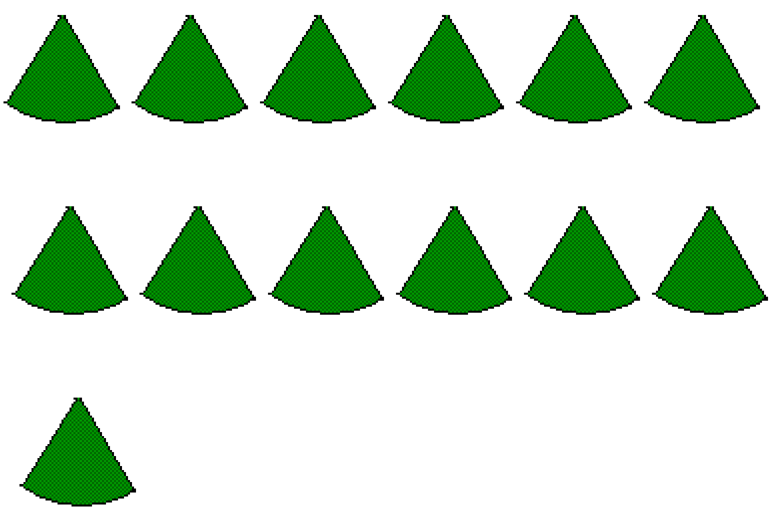
Instead of making 1s, you are trying to make as many parts as possible with the 1s.

Convert $2\frac{1}{6}$ into a mixed number.

Since you mixed number has one-sixth, you should break everything down into sixths.

$$2\frac{1}{6} = 1 + 1 + \frac{1}{6}$$

A model for this mixed number is shown below. Notice that 1 is equal to six-sixths.



All you have to do now is to count how many sixths there are.

Since there are thirteen-sixths, the improper fraction is $\frac{13}{6}$

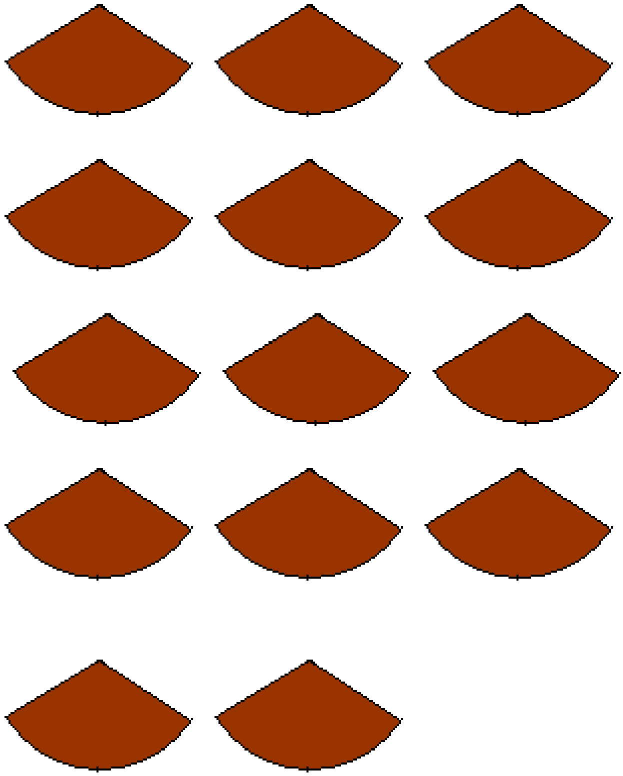
Let's do the same thing for $4\frac{2}{3}$

Since you mixed number has two-thirds, it is a good indication that you should break everything down into thirds

$$4\frac{2}{3} = 1 + 1 + 1 + 1 + \frac{2}{3}$$

A model for this mixed number is shown on the next page.

Notice that 1 is equal to three-thirds



All you have to do now is to count how many thirds there are.

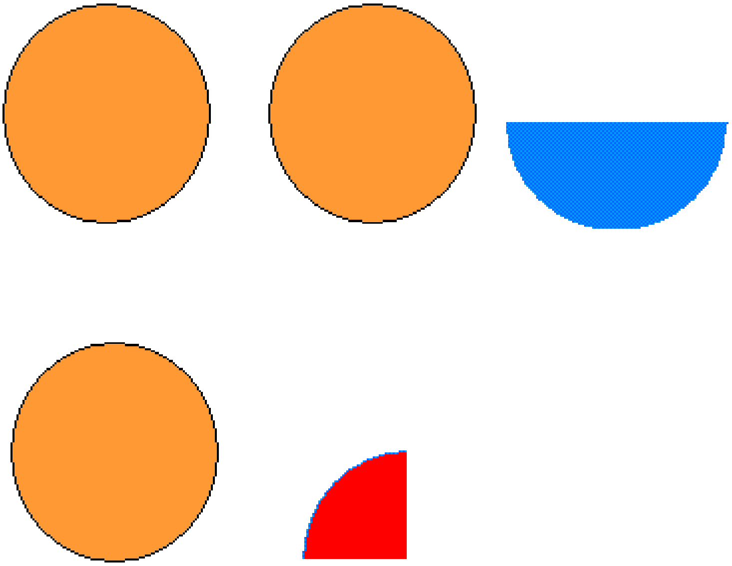
Since there are fourteen-thirds, the improper fraction is $\frac{14}{3}$

Adding mixed numbers

We can also add mixed numbers with pattern blocks.

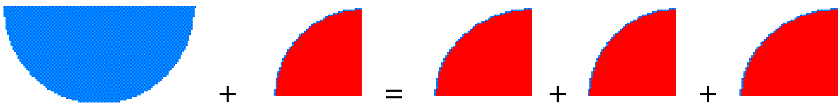
Add $2\frac{1}{2} + 1\frac{1}{4}$

First model the problem.



The three orange circles will add up to 3 because each is equal to 1

Add now the blue block to the red block. Then trade in 1 blue block for two red blocks



$$= \quad \frac{3}{4}$$

So, $2\frac{1}{2} + 1\frac{1}{4} = 3\frac{3}{4}$

You could also solve the problem this way. First, add the whole numbers. Then, add the fractions

The whole numbers are 2 and 1, so 2 + 1 = 3.

The fractions are $\frac{1}{2}$ and $\frac{1}{4}$

Since the least common multiple of 2 and 4 is 4, the common denominator is 4

$$\frac{1}{2} + \frac{1}{4} = \frac{1 \times 2}{2 \times 2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$$

Again, $2\frac{1}{2} + 1\frac{1}{4} = 3\frac{3}{4}$.

Next, we outlined a general approach we can take when adding mixed numbers

Recall mixed numbers have the following format:

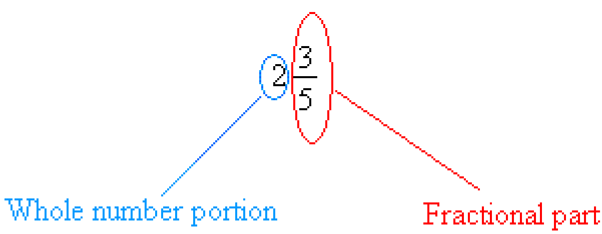
$$\textit{Whole number}\,\frac{\text{numerator}}{\text{denominator}}\ ,$$

$$\text{where}\quad \frac{\text{numerator}}{\text{denominator}}\quad \text{is a proper fraction}$$

For example, $2\,\frac{3}{5}$ is a mixed number.

When adding mixed numbers, add all whole numbers portions.

Then, add all fractional parts.



For example, $2\,\frac{1}{2} + 3\,\frac{5}{2}$

Add we said before in the lesson about mixed numbers,

$2\frac{1}{2} = 2 + \frac{1}{2}$ and $3\frac{5}{2} = 3 + \frac{5}{2}$

So, $2\frac{1}{2} + 3\frac{5}{2} = 2 + \frac{1}{2} + 3 + \frac{5}{2}$

Add the whole number portions 2 + 3 = **5**

Add the fractional parts: $\frac{1}{2} + \frac{5}{2} = \frac{1+5}{2} = \mathbf{\frac{6}{2}}$

The answer is $5\frac{6}{2}$ or $5 + \frac{6}{2}$

Notice though that $\frac{6}{2} = 3$.

Therefore, you can simplify the answer if you do 5 + 3 = 8.

In fact, you should always simplify an answer whenever possible.

Let us make a small change in the problem above.

The problem is now $2\frac{1}{2} + 3\frac{6}{2}$

Once again, 2 + 3 = 5 but $\frac{1}{2}+\frac{6}{2}=\frac{7}{2}$

The answer is $5\,\frac{7}{2}$

Notice though that the fractional part $\frac{7}{2}$ is not a proper fraction.

Furthermore, it cannot be simplified to give a whole number.

What you can do is to convert $\frac{7}{2}$ into a mixed number and add again

We know that $\frac{7}{2}=3\,\frac{1}{2}$, so $5\,\frac{7}{2} = 5 + 3\,\frac{1}{2}$

Add the whole number portions: 5 + 3 = **8**

Add the fractional parts.

There is no fractional part for 5. The only fractional part we have is $\mathbf{\frac{1}{2}}$

The answer is $8\frac{1}{2}$

you could also express the answer as an improper fraction to get

$$\frac{8\times 2+1}{2}=\frac{17}{2}$$

Example #1

$4\frac{1}{2} + 5\frac{1}{3}$

$4\frac{1}{2} + 5\frac{1}{3}$ **=** $4 + \frac{1}{2} + 5 + \frac{1}{3}$

Add the whole number portions: 4 + 5 = **9**

Add the fractional parts $\frac{1}{2} + \frac{1}{3}$

We already added $\frac{1}{2} + \frac{1}{3}$ by looking for equivalent fractions with the same denominators.

Equivalent fractions for $\frac{1}{2}$ and $\frac{1}{3}$ are respectively

$\frac{3}{6}$ and $\frac{2}{6}$ and $\frac{3}{6} + \frac{2}{6} = \mathbf{\frac{5}{6}}$

So, the answer is $9\frac{5}{6}$

Example #2

$9\frac{2}{5} + 6\frac{2}{3}$

$9\frac{2}{5} + 6\frac{2}{3}$ **=** $9 + \frac{2}{5} + 6 + \frac{2}{3}$

Add the whole number portions: 9 + 6 = **15**

Add the fractional parts $\frac{2}{5} + \frac{2}{3}$

This one two was already added before in example #1 (adding fractions with unlike denominators)**.**

$$\frac{3}{5} + \frac{2}{3} = \frac{3 \times 3}{5 \times 3} + \frac{2 \times 5}{3 \times 5} = \frac{9}{15} + \frac{10}{15} = \frac{9 + 10}{15} = \mathbf{\frac{19}{15}}$$

So, we get $15\frac{19}{15}$

Since $\frac{19}{15}$ is improper, convert it to a mixed number and add it to 15

$\frac{19}{15} = 1\frac{4}{15}$ so**,**

$$15\frac{19}{15} \mathbf{=} 15 + \frac{19}{15} = 15 + 1\frac{4}{15} = 15 + 1 + \frac{4}{15} = 16 + \frac{4}{15} = 16\frac{4}{15}$$

Thus, $9\frac{2}{5} + 6\frac{2}{3}$ **=** $16\frac{4}{15}$

Practice: Adding mixed numbers

1) $2\frac{1}{3} + 4\frac{2}{3} =$

2) $1\frac{3}{2} + 4\frac{5}{2} =$

3) $4\frac{3}{4} + 2\frac{4}{2} =$

4) $1\frac{4}{6} + 4\frac{1}{2} =$

5) $7\frac{8}{5} + 4\frac{1}{3} =$

6) $4\frac{4}{14} + 1\frac{2}{7} =$

7) $1\frac{2}{8} + 2\frac{1}{6} =$

8) $3\frac{8}{2} + 2\frac{4}{10} =$

9) $6\frac{1}{7} + 1\frac{4}{5} =$

10) $5\frac{8}{2} + 6\frac{3}{9} =$

Subtracting fractions

The rules for subtracting fractions are no different than the one for adding fractions, with the exception that instead of adding the numerators, you now subtract them.

Rule #1: ***Fractions can be subtracted only if the denominators are the same***

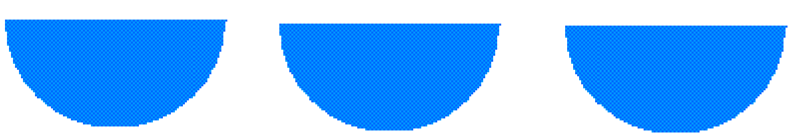
Rule #2: ***When subtracting fractions that have the same denominators, just subtract the numerators.***

We will illustrate with examples:

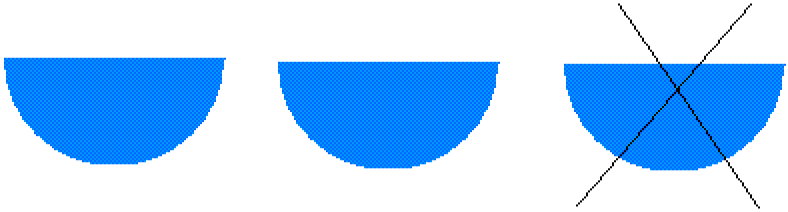
Example #1: $\frac{3}{2} - \frac{1}{2} = \frac{3-1}{2} = \frac{2}{2} = 1$

Illustration for example #1

First, model $\frac{3}{2}$



Then, cross out one half



The answer is two-halves or 1.

Example #2:

$$\frac{7}{3}-\frac{2}{3}=\frac{7-2}{3}=\frac{5}{3}$$

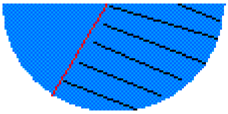
Example #3:

$$\frac{1}{2}-\frac{1}{3}=\frac{3}{6}-\frac{2}{6}=\frac{3-2}{6}=\frac{1}{6}$$

Illustration for example #3:

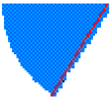
First, model half

Then, remove one-third**.** (One-third is shown with black lines)



The portion that is left is a portion that you can probably recognize

easily. We show it below. It is $\frac{1}{6}$**.**



Example #4:

$$3\,\frac{5}{2} - 2\,\frac{1}{4}$$

$$3\,\frac{5}{2} - 2\,\frac{1}{2} \quad \mathbf{=} \quad 3 + \frac{5}{2} - 2 - \frac{1}{4}$$

Subtract whole number portions: 3 – 2 **= 1**

Subtract fractional parts: $\frac{5}{2} - \frac{1}{4} = \frac{10}{4} - \frac{1}{4} = \frac{10-1}{4} = \frac{9}{4}$

We get $1\ \frac{9}{4}$

Since $\frac{9}{4}$ is an improper fractions, you could convert it into a mixed number and add it to 1

$\frac{9}{4}$ **=** $2\frac{1}{4}$ **,** so $1\ \frac{9}{4}$ **=** 1 **+** $2\frac{1}{4}$ **=** $3\ \frac{1}{4}$

Example #5:

$6\frac{1}{2} - 1\frac{1}{3}$

Subtract whole number portions: 6 – 1 = **5**

Subtract fractional parts: $\frac{1}{2} - \frac{1}{3} = \frac{3}{6} - \frac{2}{6} = \frac{3-2}{6} = \mathbf{\frac{1}{6}}$

We get $5\frac{1}{6}$

Resources:

www.basic-mathematics.com/subtracting-fractions-calculator.html

A calculator that will subtract two fractions and display the answer

http://www.basic-mathematics.com/subtracting-fractions-game.html

A game to practice subtraction of fractions

Practice: **subtracting fractions**

1) $\frac{3}{4} - \frac{2}{4} =$

2) $\frac{6}{3} - \frac{2}{3} =$

3) $\frac{1}{2} - \frac{1}{4} =$

4) $\frac{1}{3} - \frac{1}{6} =$

5) $\frac{3}{2} - \frac{2}{5} =$

6) $\frac{5}{3} - \frac{4}{7} =$

7) $4\,\frac{2}{8} - 3\,\frac{1}{8} =$

8) $5\,\frac{8}{10} - 2\,\frac{4}{10} =$

9) $8\,\frac{9}{8} - 5\,\frac{4}{16} =$

10) $4\,\frac{3}{4} - 2\,\frac{2}{5} =$

Multiplying fractions

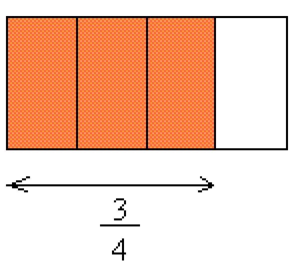
Why do we just multiply the numerators and the denominators?

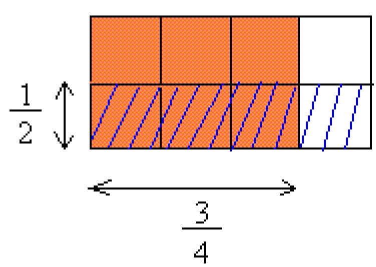
We will challenge the concept with illustrations and a real life example.

Our approach is to illustrate and then show a quick way to get the same answer

Get ready to sweat from this point on!

Look at the following illustration:





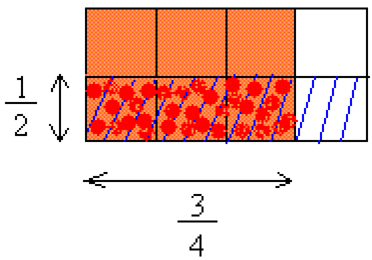
What we are trying to do is $\frac{1}{2} \times \frac{3}{4}$ and it means $\frac{1}{2}$ of $\frac{3}{4}$

First, we draw $\frac{3}{4}$ by shading the figure with the orange color and it is the figure on the left.

Second, we break the whole figure in half by shading it with blue lines.

The answer to the multiplication is where the figure is shaded twice with the orange color and the blue lines.

We show this below with red dots.



This shaded portion represents $\frac{3}{8}$ of the figure. Thus, $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

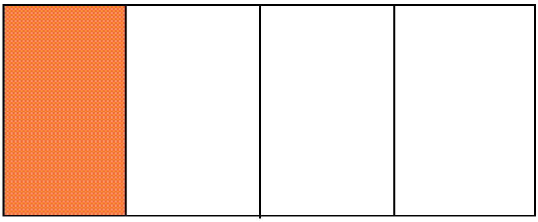
But, you can get the answer if you do $\frac{1}{2} \times \frac{3}{4} = \frac{1 \times 3}{2 \times 4} = \frac{3}{8}$

Illustration #2

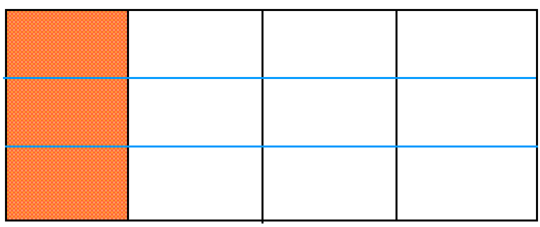
$\frac{1}{4} \times \frac{1}{3}$

Again, $\frac{1}{4} \times \frac{1}{3}$ means $\frac{1}{4}$ of $\frac{1}{3}$ or $\frac{1}{3}$ of $\frac{1}{4}$.

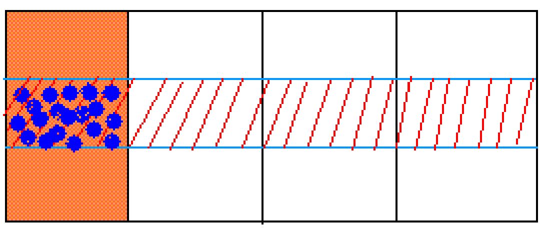
Let's model $\frac{1}{3}$ of $\frac{1}{4}$. First, you can model $\frac{1}{4}$



Then, break the same figure into 3 equal parts.



Finally, shade $\frac{1}{3}$ of the figure. This is shown with red lines



The answer to the multiplication is where the figure is shaded twice with the orange color and the red lines. We show this above with blue dots.

That area is 1 out of 12 or $\frac{1}{12}$ so $\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$

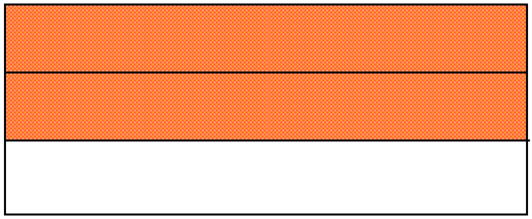
You can get the same answer if you do: $\frac{1}{4} \times \frac{1}{3} = \frac{1 \times 1}{4 \times 3} = \frac{1}{12}$

Illustration #3

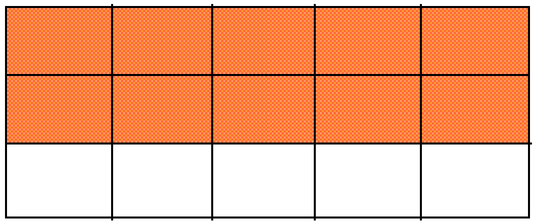
$\frac{2}{3} \times \frac{3}{5}$

Again, $\frac{2}{3} \times \frac{3}{5}$ means $\frac{3}{5}$ of $\frac{2}{3}$ or $\frac{2}{3}$ of $\frac{3}{5}$. Let's model $\frac{3}{5}$ of $\frac{2}{3}$.

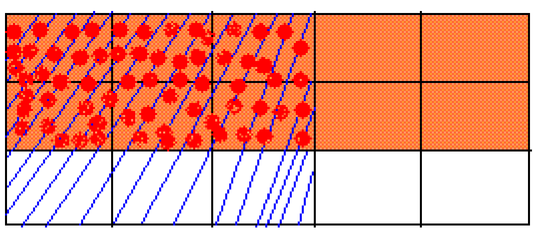
First, you can to model $\frac{2}{3}$



Then, break the same figure into 5 equal parts.



Then shade $\frac{3}{5}$ of the figure. This is shown with blue lines



The answer to the multiplication is where the figure is shaded twice with the orange color and the blue lines.

We show this above with red dots

That area is 6 out of 15 or $\frac{6}{15}$, so $\frac{2}{3} \times \frac{3}{5} = \frac{6}{15}$

You could get the same answer if you do: $\frac{2}{3} \times \frac{3}{5} = \frac{2 \times 3}{3 \times 5} = \frac{6}{15}$

Let us talk about pizzas now!

Say that you bought a medium pizza and your pizza has 8 slices.

If someone eats half of your pizza, or 4 slices, you are left with 4 slices.

The leftover can be expressed as $\frac{4}{8}$ or $\frac{1}{2}$

If you decide that you are only going to eat 1 slice out of the 4 slices remaining, you are eating $\frac{1}{4}$ of the leftover.

Remember that the leftover is $\frac{1}{2}$, so you are eating $\frac{1}{4}$ of $\frac{1}{2}$

This means $\frac{1}{4} \times \frac{1}{2}$

You can also argue that you only ate 1 slice out of 8 slices or $\frac{1}{8}$ since the pizza has 8 slices.

Thus, we can see that eating $\frac{1}{4}$ of $\frac{1}{2}$ is the same as eating $\frac{1}{8}$

Another way to get $\frac{1}{8}$ is to perform the following multiplication:

$\frac{1}{4} \times \frac{1}{2} = \frac{1 \times 1}{4 \times 2} = \frac{1}{8}$ because 1 times 1 is 1 and 4 times 2 is 8

This is an interesting result that confirm what we have been doing so far, but all you need to remember is the following:

When you multiply fractions, multiply the numerators together and then multiply the denominators together as already shown in the three illustrations above.

Multiplying fractions is thus a very straightforward concept. Suppose you are multiplying the following Fractions:

$$\frac{a}{b} \times \frac{c}{d}$$

Just multiply the numerators

Just multiply the denominators

So, $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$

Example #1:

$\frac{1}{2} \times \frac{4}{6}$

$$\frac{1}{2} \times \frac{4}{6} = \frac{1 \times 4}{2 \times 6} = \frac{4}{12}$$

Example #2

$$\frac{2}{3} \times \frac{5}{4}$$

$$\frac{2}{3} \times \frac{5}{4} = \frac{2 \times 5}{3 \times 4} = \frac{10}{12}$$

Example #3

$$\frac{4}{6} \times \frac{5}{7}$$

$$\frac{4}{6} \times \frac{5}{7} = \frac{4 \times 5}{6 \times 7} = \frac{20}{42}$$

Resources:

http://www.basic-mathematics.com/multiply-fractions-calculator.html

A calculator that will multiply two fractions and display the answer

http://www.basic-mathematics.com/multiplying-fractions-game.html

A game to see how well you can multiply fractions

Multiplying mixed numbers

Again, we will warm up the topic with good illustrations.

This time, we will play around with the pattern blocks used to add fractions.

Our approach is to illustrate and then show a quick way to get the same answer

Before we start, let us make an important point crystal clear.

2×6 means to accumulate 6 twice or $6 + 6$

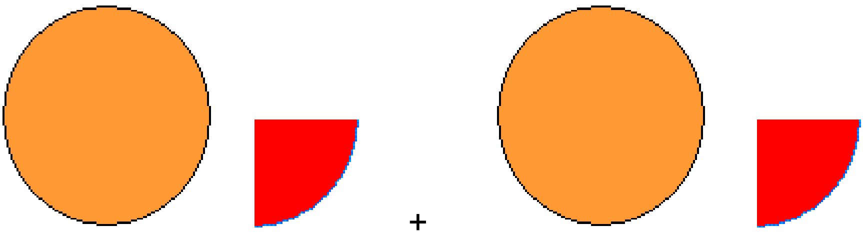
4×7 means to accumulate 7 four times or $7 + 7 + 7 + 7$

Illustration #1

$2 \times 1\frac{1}{4}$

$2 \times 1\frac{1}{4}$ means to accumulate or add $1\frac{1}{4}$ twice or $1\frac{1}{4} + 1\frac{1}{4}$

$1\frac{1}{4}$ + $1\frac{1}{4}$



2 orange blocks equal to 2 and 2 red blocks equal to $\frac{1}{2}$

$$1\frac{1}{4} + 1\frac{1}{4} = 2\frac{1}{2} = \frac{5}{2}$$

You could also get the same answer if you do

$$2 \times 1\frac{1}{4} = \frac{2}{1} \times \frac{1 \times 4 + 1}{4} = \frac{2}{1} \times \frac{5}{4} = \frac{10}{4} = \frac{10 \div 2}{4 \div 2} = \frac{5}{2}$$

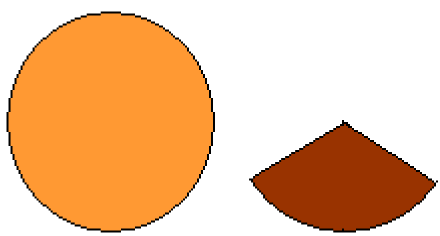
Illustration #2

$\frac{1}{2} \times 1\frac{1}{3}$

$\frac{1}{2} \times 1\frac{1}{3}$ means to accumulate $1\frac{1}{3}$ half time.

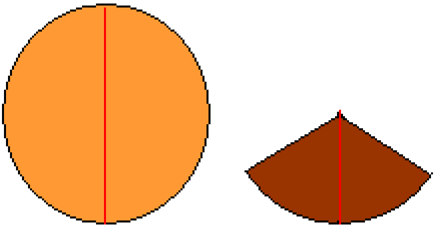
Simply lay down a representation of $1\frac{1}{3}$ and take half

$1\frac{1}{3}$ =



Take half.

This is shown with the red line



We get $\frac{1}{2}+\frac{1}{6}$

Since we can trade or swap half for three-sixths,

$$\frac{1}{2}+\frac{1}{6} = \frac{3}{6}+\frac{1}{6}=\frac{4}{6}=\frac{2}{3}$$

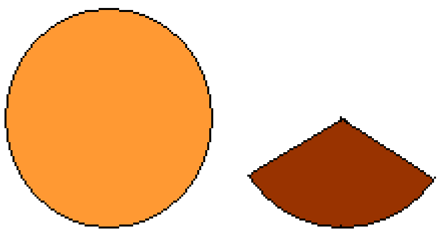
You could also get the same answer if you do

$$\frac{1}{2}\times 1\,\frac{1}{3}=\frac{1}{2}\times\frac{1\times 3+1}{3}=\frac{1}{2}\times\frac{4}{3}=\frac{4}{6}=\frac{4\div 2}{6\div 2}=\frac{2}{3}$$

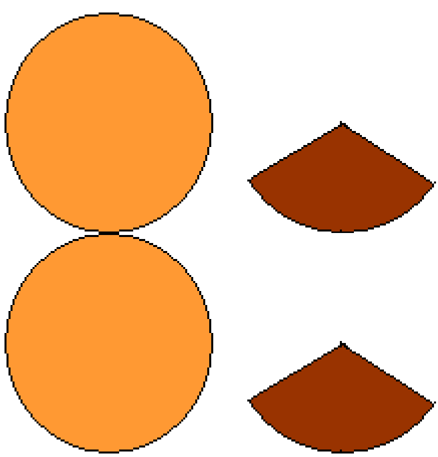
Illustration #3

$2\frac{1}{2} \times 1\frac{1}{3}$

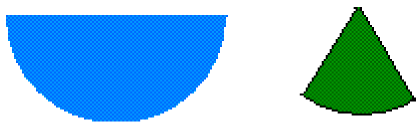
$2\frac{1}{2} \times 1\frac{1}{3}$ means to accumulate $1\frac{1}{3}$ twice and then half time.

$1\frac{1}{3}$ = 

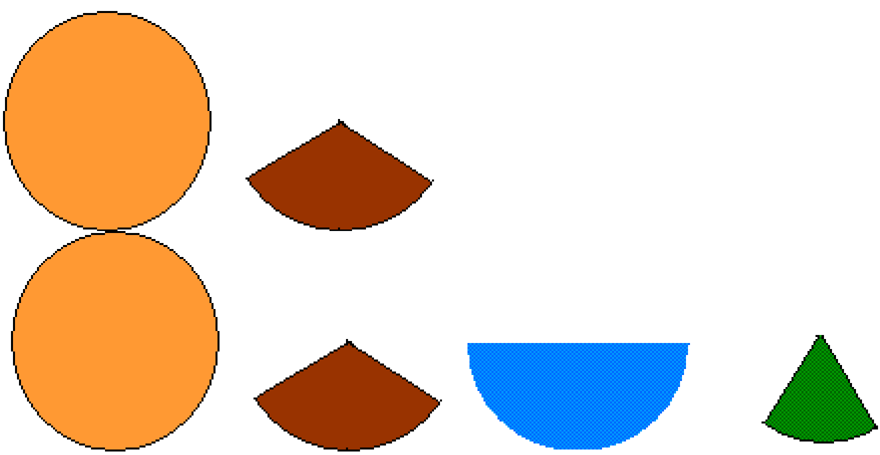
If we accumulate $1\frac{1}{3}$ twice or double the amount, we get:



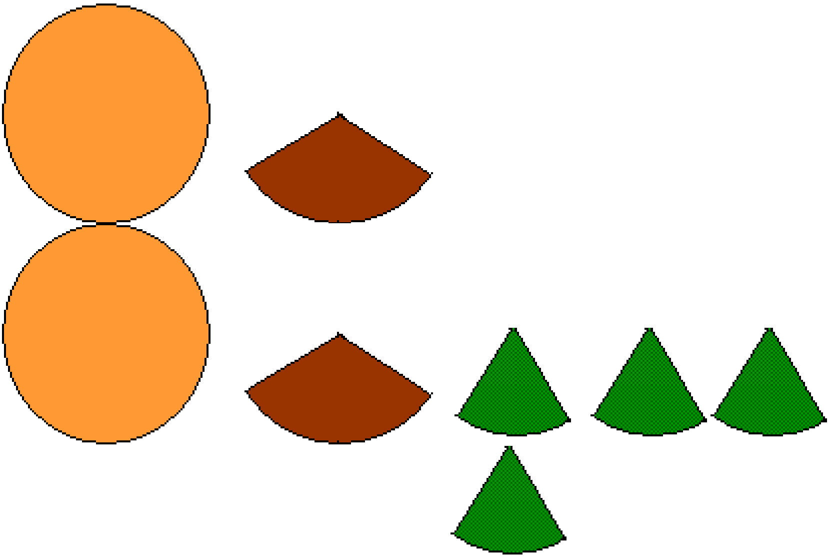
If we accumulate $1\frac{1}{3}$ half or take half, we get:



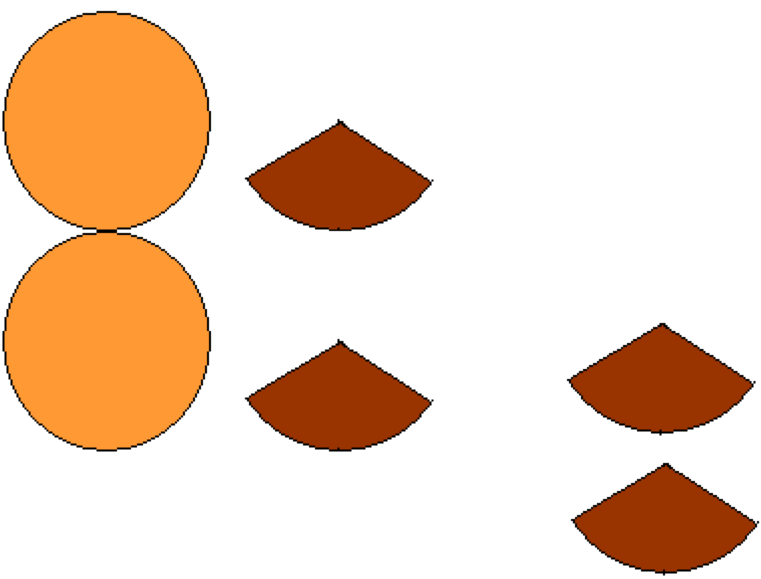
Let us put it all together



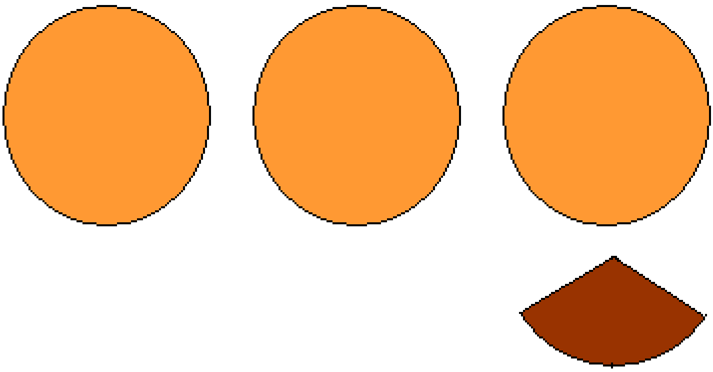
Trade in half for three-sixths



Trade in two-sixths for one-third



Trade in three-thirds for 1 whole



So, $2\frac{1}{2} \times 1\frac{1}{3} = 3\frac{1}{3}$

You could also get the answer if you do

$$2\frac{1}{2} \times 1\frac{1}{3} = \frac{2 \times 2 + 1}{2} \times \frac{1 \times 3 + 1}{3} = \frac{5}{5} \times \frac{4}{3} = \frac{20}{6} = \frac{20 \div 2}{6 \div 2} = \frac{10}{3} = 3\frac{1}{3}$$

In practice, to multiply mixed numbers, convert the mixed numbers into improper fractions before performing multiplication

3) $2\frac{3}{5} \times 4\frac{1}{4}$

$$2\frac{3}{5} \times 4\frac{1}{4} =$$

$$\frac{2 \times 5 + 3}{5} \times \frac{4 \times 4 + 1}{4} = \frac{10 + 3}{5} \times \frac{16 + 1}{4} = \frac{13}{5} \times \frac{17}{4} = \frac{13 \times 17}{5 \times 4} = \frac{221}{20}$$

4) $4\frac{2}{3} \times 1\frac{3}{2}$

$$4\frac{2}{3} \times 1\frac{3}{2} = \frac{4 \times 3 + 2}{3} \times \frac{1 \times 2 + 3}{2} = \frac{12 + 2}{3} \times \frac{2 + 3}{2} = \frac{14}{3} \times \frac{5}{2} = \frac{70}{5}$$

Practice: Multiplying fractions

1) $\frac{2}{3} \times \frac{3}{4} =$

2) $\frac{1}{4} \times \frac{2}{5} =$

3) $\frac{3}{2} \times \frac{4}{2} =$

4) $\frac{4}{6} \times \frac{1}{2} =$

5) $1\frac{8}{5} \times 4\frac{1}{3} =$

6) $3\frac{6}{7} \times 2\frac{6}{7} =$

7) $5\frac{1}{8} \times 7\frac{1}{6} =$

8) $\frac{8}{10} \times \frac{4}{2} =$

9) $\frac{6}{8} \times \frac{4}{5} =$

10) $4\frac{8}{2} \times 3\frac{3}{9} =$

Dividing fractions

Just like multiplying fractions, we can illustrate the process of dividing fractions with patterns blocks.

Let's start with an easy example. Our approach is to illustrate and then show a quick way to get the same answer

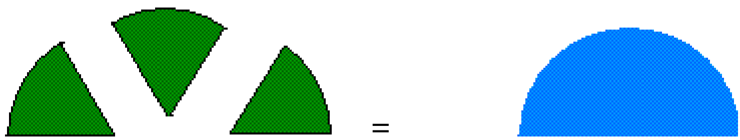
Illustration #1:

$$\frac{1}{2} \div \frac{1}{6} \qquad\qquad \frac{1}{2} \div \frac{1}{6} = \frac{\frac{1}{2}}{\frac{1}{6}}$$

The problem above means how many times $\frac{1}{6}$ will fit into $\frac{1}{2}$

Get a 3 green blocks and try putting them together as show below.

You should notice that it is going to equal the blue block.



So, 1 green block or one-sixth will fit into the blue block or half 3 times.

$$\frac{1}{2} \div \frac{1}{6} = \frac{\frac{1}{2}}{\frac{1}{6}} = 3$$

You could get the same answer if you do

$$\frac{1}{2} \div \frac{1}{6} = \frac{\frac{1}{2}}{\frac{1}{6}} = \frac{1}{2} \times \frac{6}{1} = \frac{1 \times 6}{2 \times 1} = \frac{6}{2} = 3$$

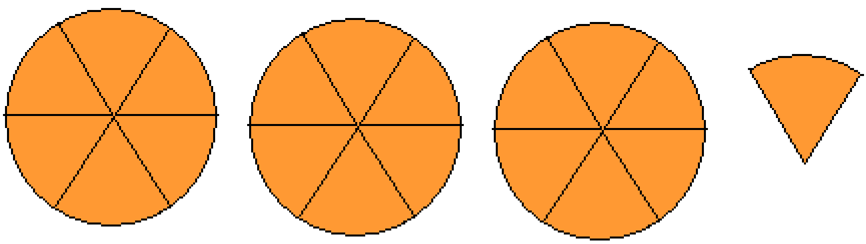
Notice that to get the same answer we change the operation into multiplication and we change $\frac{1}{6}$ to $\frac{6}{1}$

Illustration #2:

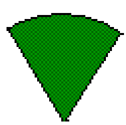
$$3\frac{1}{6} \div \frac{1}{6}$$

The problem above means how many times $\frac{1}{6}$ will fit into $3\frac{1}{6}$

First, model $3\frac{1}{6}$



Just count how many sixths there are. I see 6 + 6 + 6 + 1 = 19

So, 1 green or block or  will fit into $3\frac{1}{6}$ 19 times.

$3\frac{1}{6} \div \frac{1}{6} = 19$

You can get the same answer if you do the following:

$$3\frac{1}{6} \div \frac{1}{6} = \frac{3 \times 6 + 1}{6} \div \frac{1}{6} = \frac{19}{6} \div \frac{1}{6} = \frac{19}{6} \times \frac{6}{1} = \frac{114}{6} = 19$$

Again, notice that $\frac{1}{6}$ was changed into $\frac{6}{1}$ and the division sign was changed to a multiplication. This is what we always do.

We explain why later!

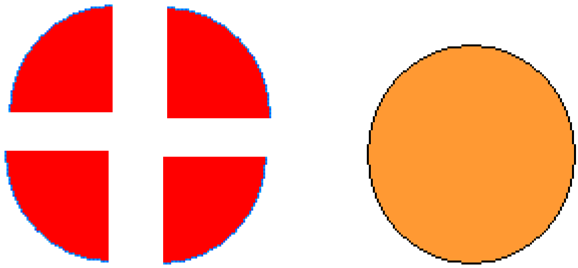
Illustration #3:

$$2 \div \frac{1}{4} \qquad\qquad 2 \div \frac{1}{4} = \frac{2}{\frac{1}{4}}$$

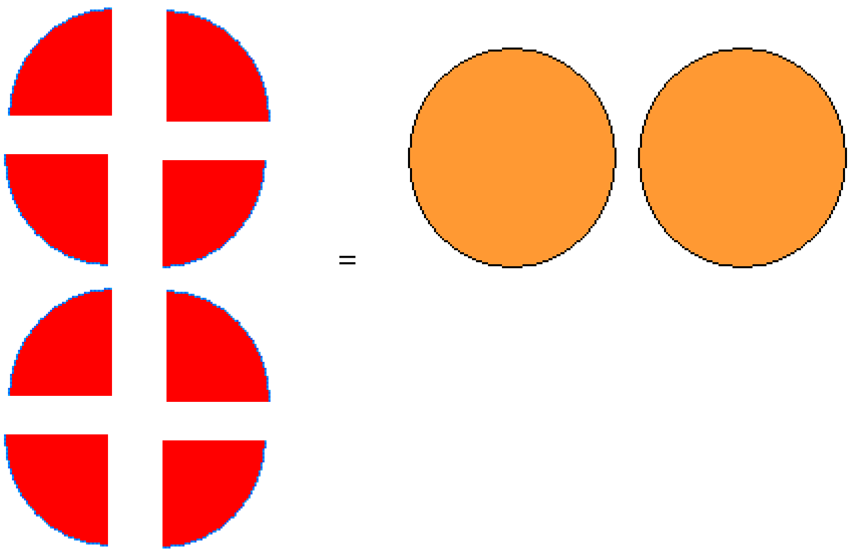
The problem above means how many times $\frac{1}{4}$ will fit into 2

Get a 4 red blocks and try putting them together as shown below.

You should notice that it is going to equal the orange block or 1.



We need then 8 red blocks to make 2 orange blocks.



So, 1 red block or one-fourth will fit into 2 orange blocks 8 times.

$$2 \div \frac{1}{4} = \frac{2}{\frac{1}{4}} = 8$$

You could get the same answer if you do

$$2 \div \frac{1}{4} = \frac{2}{\frac{1}{4}} = \frac{\frac{2}{1}}{\frac{1}{4}} = \left(\frac{2}{1}\right) \times \left(\frac{4}{1}\right) = \frac{2 \times 4}{1 \times 1} = \frac{8}{1} = 8$$

Notice again that to get the same answer we change the operation into multiplication and we change $\frac{1}{4}$ to $\frac{4}{1}$

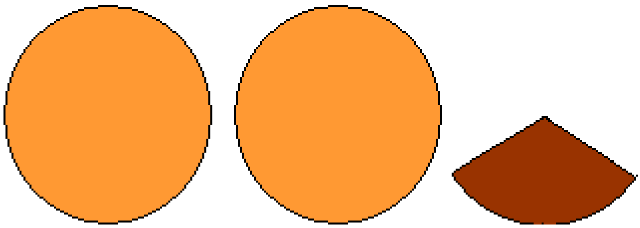
Notice also that we change 2 into $\frac{2}{1}$ since $2 = \frac{2}{1}$

Illustration #4:

$$2\frac{1}{3} \div 2 \qquad\qquad 2\frac{1}{3} \div 2 = \frac{2\frac{1}{3}}{2}$$

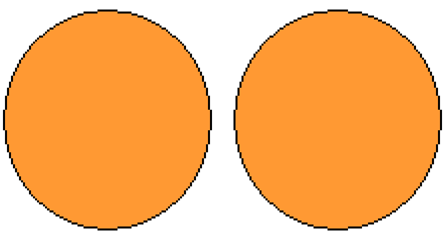
The problem above means how many times 3 will fit into $2\frac{1}{3}$

First, model $2\frac{1}{3}$. This is shown on the next page



$= \quad 2\frac{1}{3}$

Then model 2. This is shown below:



$= 2$

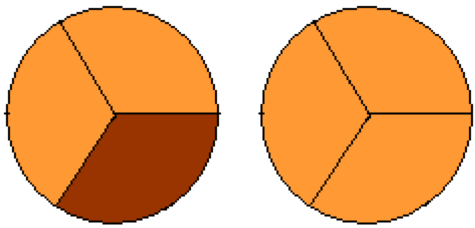
It is not hard to see that 2 fit into 2 once.

So, we have part of the answer.

However, it is complicated to see how many times 2 will fit into $\frac{1}{3}$

Our strategy will be to put $\frac{1}{3}$ or the brown block inside one of the two circles.

Then, you can see that 2 is made up of 6 brown blocks



Since 2 is bigger than $\frac{1}{3}$, the amount of times 2 will fit into $\frac{1}{3}$ is a fraction of 2

How much of 2 do I need to take to get $\frac{1}{3}$?

You need to take one-sixth of 2 or $\frac{1}{6}$ of 2.

Thus, 2 can fit into one-third $\frac{1}{6}$ time.

Putting everything together, 2 can fit into $2\frac{1}{3}$ once and one-sixths time or $1\frac{1}{6}$ time

$$2\frac{1}{3} \div 2 = \frac{2\frac{1}{3}}{2} = 1\frac{1}{6}$$

You could get the same answer if you convert $2\frac{1}{3}$ into an improper fraction and then do the same thing you did in illustration #1 and #2

$$2\frac{1}{3} \div 2 = \frac{2 \times 3 + 1}{3} \div \frac{2}{1} = \frac{7}{3} \div \frac{2}{1} = \frac{7}{3} \times \frac{1}{2} = \frac{7}{3} = 1\frac{1}{6}$$

There is an easier way to handle the same problem.

$\frac{6}{2}$ means to break 6 down into 2 equal groups.

These two equal groups are 3 and 3

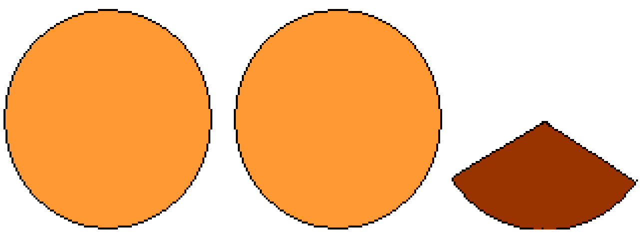
$\frac{8}{4}$ means to break 8 down into 4 equal groups.

These four equal groups are 2, 2, 2, and 2

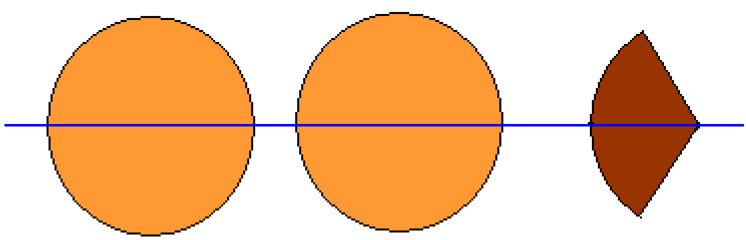
By the same token $2\frac{1}{3} \div 2$ means to break $2\frac{1}{3}$ into two equal groups or take half of everything.

The process is shown below:

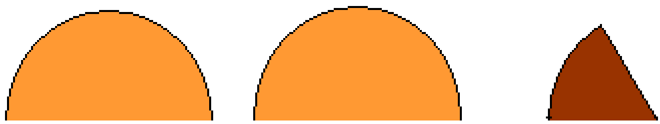
First, model $2\frac{1}{3}$



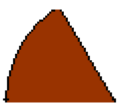
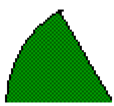
Then, draw the blue lines below that will break everything in half.



Pull out half



The two orange halves is equal to 1 and the resulting brown piece has the same size as the green piece.

 =  = $\frac{1}{6}$

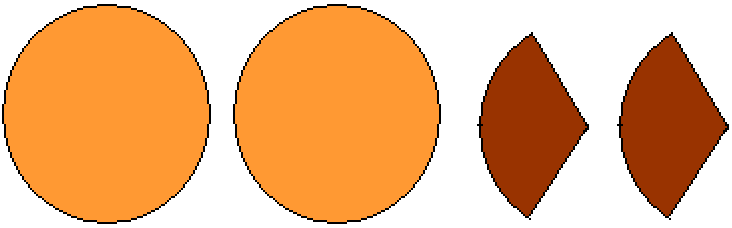
We get 1 and $\frac{1}{6}$ or $1\frac{1}{6}$

We can use the latter technique to solve $2\frac{2}{3} \div 4$

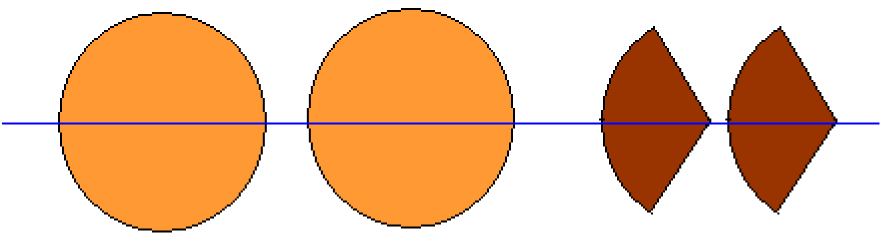
Illustration #5:

First model $2\frac{2}{3} \div 4$

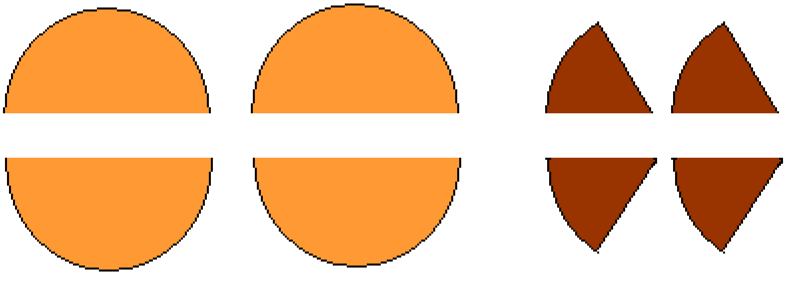
First, model $2\frac{2}{3}$



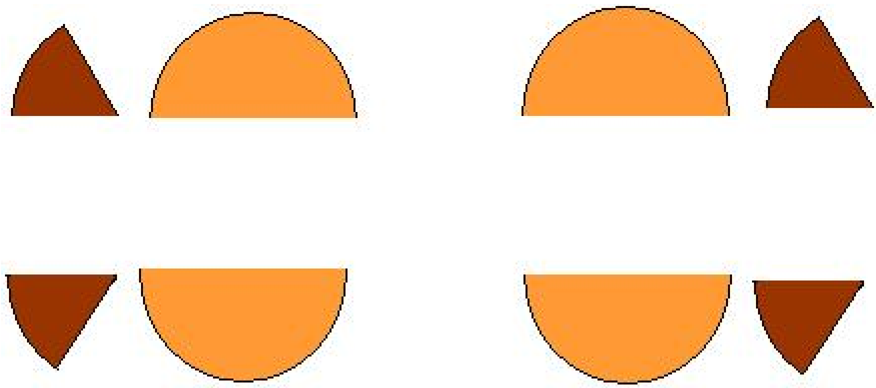
Then, draw the blue lines below that will break everything in half.



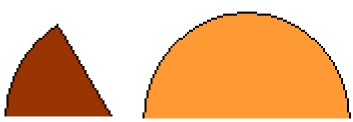
Break down and make 4 equal groups



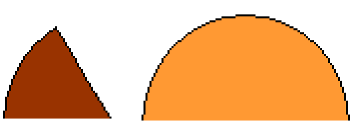
The 4 equal groups are shown below:



Take 1 group.



Trade half for three-sixths



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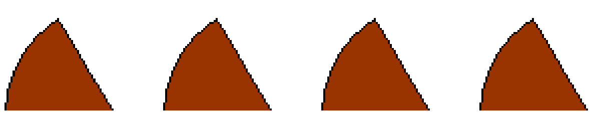

$= \frac{4}{6} = \frac{2}{3}$

Once again, you can get the same answer if you do the following:

$$2\frac{2}{3} \div 4 = \frac{2 \times 3 + 2}{3} \div \frac{4}{1} = \frac{8}{3} \div \frac{4}{1} = \frac{8}{3} \times \frac{1}{4} = \frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$$

Dividing fractions is also a very straightforward concept.

Suppose you are dividing the following fractions:

$$\frac{a}{b} \div \frac{c}{d}$$

Invert $\frac{c}{d}$ by writing it as $\frac{d}{c}$ and times it by $\frac{a}{b}$.

Thus, $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$

Now, why do we invert $\frac{c}{d}$? It is a good question.

To make this clear, we will use common denominator.

This way we will view division of fractions as an extension of whole-number division.

For example, suppose you are performing this division: $\frac{9}{2} \div \frac{1}{2}$

This answer can be obtained by asking yourself the question:

How many $\frac{1}{2}$ are in $\frac{9}{2}$?

Since $\frac{9}{2} = 9 \times \frac{1}{2}$, there are nine $\frac{1}{2}$ in $\frac{9}{2}$

You could have also asked yourself this question:

How many 1s are in 9 or what is $\frac{9}{1}$?

Since there are nine 1s in 9, there are also nine $\frac{1}{2}$ in $\frac{9}{2}$.

Similarly, if we perform the following division: $\frac{16}{3} \div \frac{4}{3}$,

you can just ask yourself how many 4s are there in 16 or what is $\frac{16}{4}$?

Since $\frac{16}{4}$ = 4, there are four 4s in 16, so there are four $\frac{4}{3}$ in $\frac{16}{3}$

Notice that the answer can be found simply by dividing the numerators in the correct order(This means to divide the numerator on the left by numerator on the right)

In the case of $\frac{7}{4} \div \frac{5}{4}$, we can ask ourselves how many $\frac{5}{4}$ are there in $\frac{7}{4}$?

It means the same to ask ourselves how many 5s are there in 7 or what is $\frac{7}{5}$?

Since $\frac{7}{5}$ does not give a whole number, it is perfectly ok to express the answer as a fraction and say that $\frac{7}{5}$ is the answer.

In general, when two fractions have the same denominator $\frac{a}{b} \div \frac{c}{b} = \frac{a}{c}$

If the fractions have different denominator, rewrite the fractions so they have the same denominator before using the formula above.

For example, $\frac{2}{3} \div \frac{4}{5}$

$$\frac{2}{3} \div \frac{4}{5} = \frac{2 \times 5}{3 \times 5} \div \frac{4 \times 3}{5 \times 3} = \frac{10}{15} \div \frac{12}{15} = \frac{10}{12}$$

In general, $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bd} \div \frac{bc}{bd} = \frac{ad}{bc}$

Notice also that $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$ and $\frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$

Note: When 2 quantities are equal to the same thing, they are equal to each other

$\frac{a}{b} \div \frac{c}{d}$ and $\frac{a}{b} \times \frac{d}{c}$ are equal to the same thing $\frac{ad}{bc}$

so $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$

Thus, when dividing fractions, the procedure is to invert the divisor and multiply by the dividend.

There is something important that you should be aware of also.

Here it is:

$$\frac{a}{b} \div \frac{c}{d} \quad = \quad \frac{\frac{a}{b}}{\frac{c}{d}}$$

The left side $\frac{a}{b} \div \frac{c}{d}$ means the same thing as $\frac{\frac{a}{b}}{\frac{c}{d}}$

It is just a matter of notation or a different way of writing the same thing.

Example #1:

$$\frac{2}{3} \div \frac{4}{5}$$

$$\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4} = \frac{10}{12}$$

It means the same if you had $\dfrac{\frac{2}{3}}{\frac{4}{5}}$

$$\frac{\frac{2}{3}}{\frac{4}{5}} = \frac{2}{3} \times \frac{5}{4} = \frac{10}{12}$$

Example #2:

$$\frac{1}{2} \div \frac{1}{6}$$

$$\frac{1}{2} \div \frac{1}{6} = \frac{1}{2} \times \frac{6}{1} = \frac{6}{2} = 3$$

To divide mixed numbers, convert the mixed numbers into improper fractions before performing division.

Example #3:

$$2\frac{3}{5} \div 4\frac{1}{4}$$

$$2\frac{3}{5} \div 4\frac{1}{4} \quad =$$

$$\frac{2\times 5+3}{5} \times \frac{4\times 4+1}{4} = \frac{10+3}{5} \div \frac{16+1}{4} = \frac{13}{5} \div \frac{17}{4} = \frac{13}{5} \times \frac{4}{17} = \frac{52}{85}$$

Example #4:

$$4\frac{2}{3} \times 1\frac{3}{2}$$

$$4\frac{2}{3} \div 1\frac{3}{2} \quad =$$

$$\frac{4\times 3+2}{3} \div \frac{1\times 2+3}{2} = \frac{12+2}{3} \div \frac{2+3}{2} = \frac{14}{3} \div \frac{5}{2} = \frac{14}{3} \times \frac{2}{5} = \frac{28}{15}$$

Practice: dividing fractions

1) $\frac{1}{3} \div \frac{3}{2} =$

2) $\frac{2}{4} \div \frac{1}{6} =$

3) $\frac{5}{3} \div \frac{1}{8} =$

4) $\frac{4}{2} \div \frac{1}{3} =$

5) $\frac{5}{5} \div \frac{4}{10} =$

6) $\frac{6}{2} \div \frac{1}{3} =$

7) $2\frac{1}{8} \div \frac{2}{4} =$

8) $\frac{5}{10} \div 4\frac{3}{5} =$

9) $1\frac{3}{4} \div 3\frac{2}{1} =$

10) $2\frac{7}{2} \div 2\frac{3}{2} =$

Answers: Introduction to fractions

1) $\frac{2}{6}$ 2) $\frac{1}{6}$ 3) $\frac{3}{4}$ 4) $\frac{2}{16}$

Answers: Equivalent fractions

1) $\frac{2}{8}$ and $\frac{1}{4}$ 2) $\frac{2}{8}$ and $\frac{1}{4}$ 3) $\frac{10}{12}$ and $\frac{20}{24}$ 4) $\frac{8}{20}$ and $\frac{16}{40}$

Answers: Simplifying fractions

1) $\frac{1}{3}$ 2) $\frac{1}{2}$ 3) $\frac{1}{3}$ 4) $\frac{1}{2}$ 5) $\frac{1}{4}$) $\frac{1}{3}$ 7) $\frac{1}{3}$ 8) $\frac{1}{8}$

Answers: Renaming mixed numbers and improper fractions

Exercise A:

1) $3\frac{4}{5}$ 2) $5\frac{2}{6}$ 3) $5\frac{2}{4}$ 4) $7\frac{1}{2}$ 5) $3\frac{1}{3}$ 6) $1\frac{4}{5}$

Exercise B:

1) $\frac{11}{5}$ 2) $\frac{14}{4}$ 3) $\frac{5}{3}$ 4) $\frac{13}{2}$ 5) $\frac{31}{3}$ 6) $\frac{17}{5}$

Answers: Comparing fractions

1) $\frac{2}{3} < \frac{3}{4}$ 2) $\frac{3}{4} > \frac{1}{2}$ 3) $\frac{3}{7} < \frac{4}{6}$ 4) $\frac{2}{5} < \frac{3}{6}$ 5) $\frac{3}{5} < \frac{3}{4}$ 6) $\frac{6}{7} < \frac{7}{8}$

Answers: Adding fractions

Practice: adding fractions with the same denominator

1) $\frac{4}{2} = 2$ 2) $\frac{7}{3}$ 3) $\frac{9}{4}$ 4) $\frac{5}{2}$ 5) $\frac{5}{5} = 1$ 6) $\frac{12}{4} = 3$

7) $\frac{2}{8}$ 8) $\frac{14}{12}$ 9) $\frac{9}{10}$ 10) $\frac{9}{9} = 1$

Practice: **Adding fractions with different denominators**

1) $\frac{17}{12}$ 2) $\frac{9}{10}$ 3) $\frac{11}{4}$ 4) $\frac{7}{6}$ 5) $\frac{29}{15}$ 6) $\frac{8}{14}$

7) $\frac{20}{48} = \frac{5}{12}$ 8) $\frac{44}{10}$ 9) $\frac{33}{35}$ 10) $\frac{78}{18}$

Practice: **Adding mixed numbers**

1) $6\frac{3}{3} = 7$ 2) $5\frac{8}{2} = 9$ 3) $6\ \frac{11}{4}$ 4) $5\ \frac{7}{6}$ 5) $11\ \frac{29}{15}$

6) $5\ \frac{8}{14}$ 7) $3\ \frac{20}{48}$ 8) $5\ \frac{44}{10}$ 9) $7\ \frac{33}{35}$ 10) $11\ \frac{78}{18}$

Answers: Subtracting fractions

1) $\frac{1}{4}$ 2) $\frac{4}{3}$ 3) $\frac{1}{4}$ 4) $\frac{1}{6}$ 5) $\frac{11}{10}$ 6) $\frac{23}{21}$

7) $1\ \frac{1}{8}$ 8) $3\ \frac{4}{10}$ 9) $3\ \frac{14}{16}$ 10) $2\ \frac{7}{10}$

Answers: Multiplying fractions

1) $\frac{6}{12}$ 2) $\frac{2}{20}$ 3) $\frac{12}{4} = 3$ 4) $\frac{4}{12}$ 5) $\frac{169}{15}$ 6) $\frac{540}{49}$

7) $\frac{68}{16}$ 8) $\frac{25}{230}$ 9) $\frac{7}{20}$ 10) $\frac{22}{14}$

Answers: Dividing fractions

1) $\frac{2}{9}$ 2) $\frac{12}{4}$ 3) $\frac{40}{3}$ 4) $\frac{12}{2} = 6$ 5) $\frac{50}{20} = \frac{5}{2}$ 6) $\frac{18}{2} = 9$

7) $\frac{68}{16}$ 8) $\frac{25}{230}$ 9) $\frac{7}{20}$ 10) $\frac{22}{14}$