Tough Algebra Problems Solved!

1.

The cost of petrol rises by 2 cents a liter. last week a man bought 20 liters at the old price. This week he bought 10 liters at the new price. Altogether, the petrol costs $9.20.What was the old price for 1 liter?

Solution

Trick: **cost of gas last week + cost of gas this week = $9.20**

Let x be the old price.

2 cents = 0.02 dollar. Let x + 0.02 be the new price.

Last week, it cost the man 20x. This week, it cost the man 10(x + 0.02)

$$20x + 10(x + 0.02) = 9.20$$

$$20x + 10x + 0.2 = 9.20$$

$$30x + 0.2 = 9.20$$

$$30x + 0.2 - 0.2 = 9.20 - 0.2$$

$$30x = 9$$

$$\frac{30}{30}x = \frac{9}{30}$$

$$x = 0.30$$

The old price for 1 liter was 0.30 and the new price is 0.30 + 0.02 = 0.32

Check the answer

0.30 times 20 = 6

0.32 times 10 = 3.2

Indeed 6 + 3.2 = 9.2 dollars

2.

Teachers divided students into groups of 3. Each group of 3 wrote a report that had 9 pictures in it. The students used 585 pictures altogether. How many students were there in all?

Trick #1: Number of groups times number of students in each group = total number of students.

Number of students in each group = 3
Number of groups = x

Trick #2: Number of groups times number of pictures in each group = 585

Using trick #2, x times 9 = 585 or 9x = 585

9x = 585

$$\frac{9}{9}x = \frac{585}{9}$$

x = 65

Number of groups = 65

Using trick #1,total students = 65 times 3 = 195

The class has 195 students.

3.

Vera and Vikki are sisters. Vera is 4 years old and Vikki is 13 years old. What age will each sister be when Vikki is twice as old as Vera?

Trick : Let x be the numbers of years that will be added to each sister when Vikki is twice as old as Vera.

In x years, Vera will be x + 4 years old and Vikki will be x + 13 years old.

Vikki will be twice as old.

$X + 13 = 2\ (x + 4)$

$x + 13 = 2x + 8$

$x + 13 - 8 = 2x + 8 - 8$

$x + 5 = 2x$

$x - x + 5 = 2x - x$

$5 = x$

In 5 years, Vera will be x + 4 or 5 + 4 = 9

In 5 years, Vikki will be x + 13 or 5 + 13 = 18

Therefore, Vera will be 9 and Vikki will be 18 and of course 18 = 2 times 9

4.

A can do a work in 14 days and working together A and B can do the same work in 10 days. In what time can B alone do the work?

Trick #1 Time it takes A and B minus time it takes A = time it takes B

Trick #2 An entire job is represented with the number 1

Using trick #2, we can reason as shown below

A and B do entire job ________________ in 10 days

How much can be done by A and B ________________ in 1 day ?

We can rewrite as

1 _________________ 10

x _________________ 1

Cross multiply

x times 10 = 1 times 1

10x = 1

$$\frac{10}{10} x = \frac{1}{10}$$

$$x = \frac{1}{10}$$

A and B can do $\frac{1}{10}$ of the work can be done in 1 day.

By the same token if A can do the work in 14 days, it can $\frac{1}{14}$ of the work in 1 day.

Using trick #1,

times it takes B = time it takes A and B – time it takes A

$$= \frac{1}{10} - \frac{1}{14}$$

$$= \frac{1}{10}\times\frac{7}{7} - \frac{1}{14}\times\frac{5}{5}$$

$$= \frac{7}{70} - \frac{5}{70}$$

$$= \frac{2}{70}$$

$$= \frac{1}{35}$$

This means that it will take B 35 days to finish the job by himself.

5.

7 workers can make 210 pairs of cups in 6 days. How many workers are required to make 450 pairs of cups in 10 days?

Trick: Try to find the number of pairs of cup that can be produced per 1 worker per day. You will need to set up 2 proportions for each of the 2 situations.

Situation #1 : 7 workers can make 210 pairs of cups in 6 days

7 workers can produce ______________ 210 pairs of cups

1 worker ______________ x

This relationship are in proportion, so we can write

Proportion #1 $\frac{7}{1} = \frac{210}{x}$

7 times x = 210 times 1

7x = 210

$\frac{7}{7}$ x = $\frac{210}{7}$

x = 30

1 worker can produce 30 pairs of cups ________________ in 6 days

x pairs ? __________________ 1 day

This relationship are in proportion, so we can write

Proportion #2: $\frac{30}{x} = \frac{6}{1}$

6 times x = 30 times 1

6x = 30

$\frac{6}{6}$ x = $\frac{30}{6}$

x = 5

Result #1:

1 worker can produce 5 pairs of cups ________________ in 1 day

Let n be the number of workers that can produce 450 pairs of cups in 10 days

Situation #2 : n workers can make 450 pairs of cups in 10 days

n workers can produce _____________ 450 pairs of cup

1 worker _____________ x

This relationship are in proportion, so we can write

Proportion #1 $\frac{n}{1} = \frac{450}{x}$

x times n = 450 times 1

nx = 450

$\frac{n}{n}$ x = $\frac{450}{n}$

$x = \frac{450}{n}$

1 worker can produce $\frac{450}{n}$ pairs of cups ________________ in 10 days

x pairs ? ________________ 1 day

This relationship are in proportion, so we can write

Proportion #2: $\frac{\frac{450}{n}}{x} = \frac{10}{1}$

We can cross multiply so that the equation is easier to solve.

10 times x = $\frac{450}{n}$ times 1

$10x = \frac{450}{n}$

$\frac{10}{10} x = \frac{\frac{450}{n}}{10}$

$x = \frac{450}{10n}$

Result #2:

1 worker can produce $\frac{450}{10n}$ pairs of cup ________________ in 1 day

Result #1:

1 worker can produce 5 pairs of cup ____________________ in 1 day

We need to assume that each worker has the potential to produce the same amount of work per day

This means that $\frac{450}{10n}$ = 5

Multiply both sides by 10n and we get 450 = 5 times 10n

450 = 50n

Since 9 times 50 = 450, n = 9.

9 workers can make 450 pairs of cup in 10 days

6.

Ten years ago the ratio between the ages of Mohan and Suman was 3:5. 11 years hence it will be 11:16. What is the present age of Mohan?

Trick: Let x be the age now and then set up two proportions

Let x be the present age of Mohan. Let y be the age of Suman.

10 years ago, the age of Mohan was x – 10 and the age of Suman was y – 10

The ratio of Mohan's age to Suman's age 10 years ago is $\frac{x-10}{y-10}$

This ratio is also equal to $\frac{3}{5}$

$$\frac{x-10}{y-10} = \frac{3}{5}$$

Cross multiply

5 (x – 10) = 3 (y – 10)

5x – 50 = 3y – 30

5x – 3y = 50 – 30

5x – 3y = 20 equation 1

11 years hence or in the future, the age of Mohan was x + 11 and the age of Suman was y + 11

The ratio of Mohan’s age to Suman’s age 11 years hence is $\frac{x+11}{y+11}$

This ratio is also equal to $\frac{11}{16}$

$$\frac{x+11}{y+11} = \frac{11}{16}$$

Cross multiply

16 (x + 11) = 11 (y + 11)

16x + 176 = 11y + 121

16x – 11y = 121 – 176

$16x - 11y = -55$ equation 2

Put equation 1 together with equation 2 and solve it

$5x - 3y = 20$

$16x - 11y = -55$

Multiply $5x - 3y = 20$ by -16 and multiply $16x - 11y = -55$ by 5. Then add the two equations.

$$\begin{array}{r} -80x + 48y = -320 \\ 80x + -55y = -275 \\ \hline 0 \quad + -7y = -595 \end{array}$$

$y = 85$ since -7 times 85 = -595

$5x$ – 3 times 85 = 20

$5x - 255 = 20$

$5x = 275$

$x = 55$. Then, the present age of Mohan is 55.

7.

The ratio of girls to boys in class is 9 to 7 and there are 80 students in the class. How many girls are in the class?

Trick: Let x be the number of girls and let y be the number of boys. Then one of the equations is a proportion.

$$\frac{x}{y} = \frac{9}{7}$$

Cross multiply

$7x = 9y$ (1)

$x + y = 80$ (2)

Solve for y in equation 2

$x + y = 80$

$x - x + y = 80 - x$

$y = 80 - x$

Replace y in equation 1

$7x = 9(80 - x)$

$7x = 720 - 9x$

$7x + 9x = 720 - 9x + 9x$

$16x = 720$

$$\frac{16}{16}x = \frac{720}{16}$$

$x = 45$

There are 45 girls in the classroom.

8

One ounce of solution X contains only ingredients a and b in a ratio of 2:3. One ounce of solution Y contains only ingredients a and b in a ratio of 1:2. If solution Z is created by mixing solutions X and Y in a ratio of 3:11, then 2520 ounces of solution Z contains how many ounces of a?

Trick: besides the proportions below, notice that Z = x + y = 2520. You can use the proportion for solution Z and x + y = 2520 to find x and y. By the same token X = a + b and Y = a + b

Solution X: $\frac{a}{b} = \frac{2}{3}$

Solution Y: $\frac{a}{b} = \frac{1}{2}$

Solution Z: $\frac{x}{y} = \frac{3}{11}$

Use $\frac{x}{y} = \frac{3}{11}$ and cross multiply

11x = 3y   (1)

x + y = 2520   (2)

Use equation 2 to solve for x

x + y = 2520

x + y – y = 2520 – y

x = 2520 – y

Replace x in equation 1

$11\ (2520 - y) = 3y$

$27720 - 11y = 3y$

$27720 - 11y + 11y = 3y + 11y$

$27720 = 14y$

$$\frac{27720}{14} = \frac{14}{14}\, y$$

$1980 = y$

$x = 2520 - y = 2520 - 1980 = 540$

Solution Z: $x = 540$ and $y = 1980$

We saw before that X = a + b, so a + b = 540. $\frac{a}{b} = \frac{2}{3}$ is equivalent to 3a = 2b after cross multiplying.

Use a + b = 540 and 3a = 2b to find a and b

$a + b = 540$

$3a = 2b$

$a + b = 540$, so $a = 540 - b$

$3\ (540 - b) = 2b$

$1620 - 3b = 2b$

$1620 = 3b + 2b$

$1620 = 5b$

$\frac{1620}{5} = \frac{5}{5}b$

$324 = b$

$a = 540 - 324 = 216$

Solution X: $a = 216$ and $b = 324$

We saw before that $Y = a + b$, so $a + b = 1980$. $\frac{a}{b} = \frac{1}{2}$ is equivalent

to $2a = b$ after cross multiplying.

Use $a + b = 1980$ and $2a = b$ to find a and b

$a + b = 1980$

$2a = b$

$a + b = 1980$, so $a = 1980 - b$

$2(1980 - b) = b$

$3960 - 2b = b$

$3960 = 2b + b$

$3960 = 3b$

$$\frac{3960}{3} = \frac{3}{3} b$$

$1320 = b$

$a = 1980 - 1320 = 660$

Solution Y: $a = 660$ and $b = 1320$

The problem asked for the amount of ingredient a in z.

Add the amount of a in solution x and y to get the answer. Add

$660 + 216 = 876$

Therefore, 2520 of solution Z has 876 ounces of ingredient a.

9.

This week Bob puts gas in his truck when the tank was about half empty. Five days later, bob puts gas again when the tank was about three fourths full. If Bob Bought 24 gallons of gas, how many gallons does the tank hold?

Trick #1: Amount of gas Bob puts this week plus amount of gas Bob puts 5 days later = 24 gallons

Trick #2: This week, the tank was half empty, so Bob needs to add half to fill the tank

Trick #3: 5 days later, the tank was three-fourths full, so Bob needs to add one-fourth to fill the tank

Let x be the number of gallons the tank can hold

This week Bob needs to add $\frac{1}{2}$ of x or $\frac{1}{2}$ times x = $\frac{1}{2}$ x

In 5 days, Bob needs to add $\frac{1}{4}$ of x or $\frac{1}{4}$ times x = $\frac{1}{4}$ x

Using trick #1,

$$\frac{1}{2} x + \frac{1}{4} x = 24$$

$$\frac{2}{4} x + \frac{1}{4} x = 24$$

$$\frac{3}{4} x = 24$$

$$\frac{4}{3} \times \frac{3}{4} x = 24 \times \frac{4}{3}$$

$$x = \frac{24}{1} \times \frac{4}{3}$$

$$x = \frac{96}{3}$$

$$x = 32$$

The tank can hold 32 gallons of gas.

10.

A commercial airplane flying with a speed of 700 mi/h is detected 1000 miles away with a radar. Half an hour later an interceptor plane flying with a speed of 800 mi/h is dispatched. How long will it take the interceptor plane to meet with the other plane?

Trick #1: The airplanes are traveling toward each other until they meet or until they cover a distance of 1000 miles together.

Trick #2: The commercial airplane already traveled half an hour when the interceptor plane is dispatched. So the commercial airplane travels longer.

Let d1 be the distance the commercial airplane covers.

Let d2 be the distance the interceptor airplane covers.

Using trick #1, $d1 + d2 = 1000$

Let t be the time the interceptor plane travels.

Using trick #2, $t + 0.5$ is the time the commercial airplane travels.

Now, $d1 = 700 \times (t + 0.5)$ and $d2 = 800 \times t$

$700 \times (t + 0.5) + 800t = 1000$

$700t + 350 + 800t = 1000$

$1500t + 350 = 1000$

$1500t = 1000 - 350$

$1500t = 650$

$t = 0.4333$

0.433 times 60 minutes = 25.998 or 26 minutes

It took the interceptor plane 26 minutes to reach the commercial plane.

$t + 0.5 = 0.433 + 0.5 = 0.9333$

0.9333 times 60 minutes = 55.998 or 56 minutes

11.

There are 40 pigs and chickens in a farmyard. Joseph counted 100 legs in all. How many pigs and how many chickens are there?

Trick #1: Chickens have 2 legs and pigs have 4 legs.

Trick #2: 2 legs x number of chickens + 4 legs x number of pigs = 100 legs

Let y be the number of chickens, then 40 – y is the number of pigs.

using trick #2,

2 legs x y + 4 legs x (40 – y) = 100

$2y + 4(40 - y) = 100$

$2y + 160 - 4y = 100$

$-2y + 160 = 100$

$-2y = 100 - 160$

$-2y = -60$

Since -2 times 30 = -60, y = 30

There are 30 chickens and 40 – 30 or 10 pigs.

12.

The top of a box is a rectangle with a perimeter of 72 inches. If the box is 8 inches high, what dimensions will give the maximum volume?

Trick: Write down the formula for the perimeter of the top of the rectangle and solve for either w or l. Then, use the formula for volume (v = L x w x h)

P = 2L + 2 w. Since p = 72 inches, 2l + 2w = 72

Use 2L + 2w = 72 to solve for w.

$2L + 2w = 72$

$2L - 2L + 2w = 72 - 2L$

$2w = 72 - 2L$

$$\frac{2}{2}w = \frac{72}{2} - \frac{2}{2}L$$

$w = 36 - L$

Replace w and h into the volume formula.

$v = Lx(36 - L)x\ 8$

$v = 8L(36 - L)$

The equation $v = 8L(36 - L)$ is a parabola. The maximum value is between the roots of the parabola

To maximize the volume of the box, we need to find the value for L that is between the roots of the parabola.

We can find the roots by solving the parabola.

Let $v = 0$ so we can find L.

$8L(36 - L) = 0$

$8L = 0$ and $36 - L = 0$

$L = 0$ and $L = 36$

The value of L between 0 and 36 is 18.

Therefore, $L = 18$ will maximize the volume.

13.

You are raising money for a charity. Someone made a fixed donation of 500. Then, you require each participant to make a pledge of 25 dollars. What is the minimum amount of money raised if there are 224 participants.

Trick #1: Recognize that this is an arithmetic sequence.

Trick #2: Since there are many participants, try to come up with a formula that will help to do the math faster.

Let a1 be the first term of an arithmetic sequence and let d be the number that is added each time.

$a_2 = a_1 + d$

$a_3 = a_2 + d = a_1 + d + d = a_1 + 2d$

$a_4 = a_3 + d = a_1 + 2d + d = a_1 + 3d$

:
:
:
:

$a_n = a_1 + (n - 1)d$

Trick #3: 224 participants means that n = 225 since we need to add the person who made a fixed donation to the list or sequence.

$a_{225} = 500 + (225 - 1)25$

$a_{224} = 500 + 224 \times 25$

$a_{224} = 500 + 5600$

$a_{224} = 6100$

You will raise 6100 dollars.

14.

The sum of two positive numbers is 8 and the sum of their squares is 34. What are the two numbers?

$x + y = 8$ equation 1

$x^2 + y^2 = 34$ Equation 2

Use equation 1 to solve for x

$x + -x + y = 8 - x$

$y = 8 - x$

Replace $8 - x$ into equation 2

$x^2 + (8-x)^2 = 34$

$x^2 + 64 - 16x + x^2 = 34$

$x^2 + x^2 + 64 - 16x = 34$

$2x^2 - 16x + 64 = 34$

$2x^2 - 16x + 64 - 34 = 34 - 34$

$2x^2 - 16x + 30 = 0$

Divide everything by 2

$x^2 - 8x + 15 = 0$

Look for factors of 15 that will add up -8

-3 times -5 = 15 and -3 + -5 = -8

x^2 -8x + 15 = (x – 5)(x – 3) = 0

x – 5 = 0 or x = 5   x – 3 = 0 or x = 3

15.

Flying against the jet stream, a jet travels 1880 mi in 4 hours. Flying with the jet stream, the same jet travels 5820 mi in 6 hours. What is the rate of the jet in still air and what is the rate of the jet stream?

Trick: Let J be the rate of the jet in still air and let S be the rate of the jet stream. Then, realize that when the jet is flying with the jet stream,the jet stream will increase the speed of the jet. So, the rate is J + S. On the other hand, when the jet is flying against the jet stream,the jet stream will reduce the speed of the jet. So,the rate is J – S.

When the jet is flying with the jet stream, the rate is J + S

Using d = v x t, we get 5820 = (J + S)x 6 or 5820 = 6J + 6S

When the jet is flying against the jet stream, the rate is J – S

Using d = v x t, we get 1880 = (J - S)x 4 or 1880 = 4J - 4s

Use 6J + 6S = 5820 and 4J – 4S = 1880 to find J and S.

6J + 6S = 5820   equation 1

4J – 4S = 1880   equation 2

Multiply equation 1 by 4 and equation 2 by 6

24J + 24S = 23280   equation 3

24J – 24S = 11280   equation 4

Add equation 3 and equation 4 together

$48J = 34560$

$$\frac{48}{48}J = \frac{34560}{48}$$

$J = 720$

Replace J in equation 1

$6 \times 720 + 6S = 5820$

$4320 + 6S = 5820$

$6S = 5820 - 4320$

$6S = 1500$

$$\frac{6}{6}S = \frac{1500}{6}$$

$S = 250$

16.

Jenna and her friend, Khalil, are having a contest to see who can save the most money. Jenna has already saved $110 and every week she saves an additional $20. Khalil has already saved $80 and every week he saves an additional $25. Let x represent the number of weeks and y represent the total amount of money saved. Determine in how many weeks Jenna and Khalil will have the same amount of money.

Trick: Let x be the number of weeks.

Money saved = money already saved + money saved after x weeks

Money already saved by Jenna = 110

Money already saved by Khalil = 80

Total money saved by Jenna after x weeks = 110 + 20 times x = 110 + 20x

Total money saved by Khalil after x weeks = 80 + 25 times x = 80 + 25x

Jenna and Khalil will have the same amount of money when

$110 + 20x = 80 + 25x$

$110 - 80 + 20x = 80 - 80 + 25x$

$30 + 20x = 25x$

$30 + 20x - 20x = 25x - 20x$

$30 = 5x$

Since 5 times 6 = 30, x = 6

Jenny and Khalil will have the same amount of money after 6 weeks

Indeed, 110 + 20 x 6 = 110 + 120 = 230 and 80 + 25 x 6 = 80 + 150 = 230

17.

the sum of three consecutive terms of a geometric sequence is 104 and their product is 13824.find the terms.

Trick: Any term of a geometric sequence can be modeled with

x_n = a x r^{n-1} or a r^{n-1}

The second term is x_{n+1} = a x r^{n+1-1} = a r^n

The third term is x_{n+2} = a x r^{n+2-1} = a r^{n+1}

Now, we can set up two equations using the 2 conditions given in the problem.

The sum of three consecutive terms of a geometric sequence is 104

a r^{n-1} + a r^n + a r^{n+1} = 104 equation 1

The product of three consecutive terms of a geometric sequence is 13824

a r^{n-1} x a r^n x a r^{n+1} = 13824 equation 2

Let us use equation 2 to solve for a r^n

a x a x a x r^{n-1} x r^n x r^{n+1} = 13824

a^3 x $r^{n-1+n+n+1}$ = 13824

a^3 x r^{3n} = 13824

$(ar^n)^3$ = 13824

ar^n = $\sqrt[3]{13824}$

ar^n = 24

Now replace the value of ar^n into equation 1 and equation 2

a r^{n-1} + 24 + a r^{n+1} = 104

a r^{n-1} + 24 -24 + a r^{n+1} = 104 – 24

a r^{n-1} + a r^{n+1} = 80 equation 3

a r^{n-1} x 24 x a r^{n+1} = 13824 equation 2

Divide both sides of a r^{n-1} x 24 x a r^{n+1} = 13824 by 24

a r^{n-1} x a r^{n+1} = 576 equation 4

Using equation 3 and equation 4, we are just looking for two numbers whose sum is 80 and the product is 576.

576 = 72 x 8 and 72 + 8 = 80

Therefore, the 3 numbers are 8, 24, and 72

18.

The sum of the first and last of four consecutive odd integers is 52. What are the four integers?

Trick: Let 2n + 1 be the first odd integer. Then, 2n + 3 is the second, 2n + 5 is the third, and 2n + 7 is the fourth.

The sum of the first and last is 2n + 1 + 2n + 7 = 4n + 8

This sum is equal 52. So, 4n + 8 = 52

Solve 4n + 8 = 52

4n + 8 - 8 = 52 – 8

4n = 44

since 4 x 11 = 44, n = 11

The first odd integer is 2n + 1 or 2 x 11 + 1 = 22 + 1 = 23

The second odd integer is 2n + 3 or 2 x 11 + 3 = 22 + 3 = 25

The first odd integer is 2n + 5 or 2 x 11 + 5 = 22 + 5 = 27

The first odd integer is 2n + 7 or 2 x 11 + 7 = 22 + 7 = 29

19.

A health club charges a one-time initiation fee and a monthly fee. John paid 100 dollars for 2 months of membership. However, Peter paid 200 for 6 months of membership. How much will Sylvia pay for 1 year of membership?

Trick : Find the monthly fee and the one-time initiation fee. Model the problem with a system of linear equation.

2 x monthly fee + initiation fee = 100 equation 1

6 x monthly fee + initiation fee = 200 equation 2

Let m be the monthly fee and let n be the initiation fee

$$2m + n = 100$$

$$6m + n = 200$$

Use equation 1 to solve for n

$$2m + n = 100$$

$$2m - 2m + n = 100 - 2m$$

$$n = 100 - 2m$$

Replace n with 100 – 2m in equation 2

$$6m + 100 - 2m = 200$$

$$6m - 2m + 100 = 200$$

$$4m + 100 = 200$$

$$4m + 100 - 100 = 200 - 100$$

$$4m = 100$$

m = 25 since 4 times 25 = 100

Use equation 1 to find n

$$2 \times 25 + n = 100$$

$$50 + n = 100$$

$$50 - 50 + n = 100 - 50$$

$$n = 50$$

The monthly fee is 25 dollars and the initiation fee is 50 dollars.

For 1 year of membership, Sylvia will pay 25 x 12 + 50 = 300 + 50 = 350

20.

The sum of two positive numbers is 4 and the sum of their cubes is 28. What is the product of the two numbers?

Trick: Know the meaning of sum of their cubes.

The cube of x is x^3 and the cube of y is y^3

The sum of their cubes is x^3 + y^3

Now we have two equation to solve.

$x+y$ = 4 equation 1

x^3 + y^3 = 28 equation 2

Use equation 1 to solve for y

x + y = 4

x – x + y = 4 – x

y = 4 – x

Replace y in equation 2 with 4 – x

x^3 + $(4-x)^3$ = 28 equation 3

Use formula this formula $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$(4-x)^3$ = $4^3-3\times4^2\times x+3\times4\times x^2-x^3$

= $64-48x+12x^2-x^3$

Replace $64-48x+12x^2-x^3$ into equation 3

x^3 + $64-48x+12x^2-x^3$ = 28

x^3 - x^3 + $64-48x+12x^2$ = 28

$64-48x+12x^2$ = 28

$64-28-48x+12x^2$ = 28 – 28

$36-48x+12x^2$ = 0

Divide both sides of the equation by 12

$3-4x+x^2$ = 0

x^2-4x+3 = 0

We just need to solve x^2-4x+3 = 0

First factor x^2-4x+3

Just look for factors of 3 that will add up to -4

3 = -3 times -1 and -3 + -1 = -4. Therefore, the factors are -3 and -1

$x^2 - 4x + 3$ = (x – 1)(x – 3)

(x – 1)(x – 3) = 0

x – 1 = 0 and x – 3 = 0

x = 1 and x = 3

Indeed, 1 + 3 = 4 and $1^3 + 3^3 = 1 + 27 = 28$

The answer to the question is 1 times 3 = 3 since it asked to find th product of the numbers.

21.

A man selling computer parts realizes that when he sells 16 computer parts, his earning is $1700. When he sells 56 computer parts, his earning is $4300. What will the earning be if the man sells 30 computers parts?

Trick: Try to model the problem with a linear equation: y = mx + b

m is the slope and b is the y-intercept.

To find m, use m = $\frac{y_1 - y_2}{x_1 - x_2}$

($x_{1,}, y_1$) = (56, 4300) and ($x_{2,}, y_2$) = (16, 1700)

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

$$= \frac{4300 - 1700}{56 - 16}$$

$$= \frac{2600}{40}$$

= 65

y = mx + b = 65x + b

Use y = 65x + b and either (56, 4300) or (16, 1700) to find b.

Use (16, 1700). In this case, x = 16 and y = 1700

1700 = 65 x 16 + b

1700 = 1040 + b

1700 – 1040 = 1040 – 1040 + b

660 = b

The man's business can be modeled with the equation y = 65x + 660

When the man sells 30 computers, x = 30.

y = 65 x 30 + 660 = 1950 + 660 = 2610

The many earns 2610 dollars when he sells 30 computers.

22.

A man has 15 coins in his pockets. These coins are dimes and quarters that add up 2.4 dollars. How many quarters and how many dimes does the man have?

Trick: number of dimes x value of 1 dime + number of quarter x value of 1 quarter = 2.4

Let y be the number of dimes, then 15 – y will be the number of quarters.

Now, let us use the trick and keep in mind that 1 dime = $0.10 and 1 quarter = $0.25

$y \times 0.10 + (15 - y)\ 0.25 = 2.4$

$0.10y + 3.75 - 0.25y = 2.4$

$0.10y - 0.25y + 3.75 = 2.4$

$-0.15y + 3.75 = 2.4$

$-0.15y + 3.75 - 3.75 = 2.4 - 3.75$

$-0.15y = 2.4 - 3.75$

$-0.15y = -1.35$

Divide both sides by -0.15.

$$\frac{-0.15}{-0.15}y = \frac{-1.35}{-0.15}$$

$y = 9$

There are 9 dimes and 15 – 9 quarters or 6 quarters.

23.

The lengths of the sides of a triangle are in the ratio 4:3:5. Find the lengths of the sides if the perimeter is 18 inches.

Trick: Understand the meaning of 4:3:5.

Let x, y and z be the numbers in the order given in the ratio.

Then, $\frac{x}{y} = \frac{4}{3}$ $\frac{x}{z} = \frac{4}{5}$ $\frac{y}{z} = \frac{3}{5}$

Perimeter = x + y + z or 18 = x + y + z

Using $\frac{x}{y} = \frac{4}{3}$ and $\frac{x}{z} = \frac{4}{5}$, 4y = 3x and 4z = 5x

multiply both sides of the equation 18 = x + y + z by 4

18 x 4 = 4x + 4y + 4z

72 = 4x + 4y + 4z

Since 4y = 3x and 4z = 5x, we can replace 4y and 4z into the equation above.

72 = 4x + 3x + 5x

72 = 12x

Since 12 times 6 = 72, x = 6

4y = 3x

4y = 3 times 6

4y = 18

y = 18/4 = 4.5

6 + 4.5 + z = 18

10.5 + z = 18

z = 18 – 10.5

z = 7.5

24.

The ratio of base to height of a equilateral triangle is 3:4. If the area of the triangle is 6, what is the perimeter of the triangle?

Trick: Understand the meaning of 3:4 and write a proportion for the problem.

Let b be the base and let h be the height, then $\frac{b}{h}$ = $\frac{3}{4}$

Using $\frac{b}{h}$ = $\frac{3}{4}$, we can cross multiply to get 3h = 4b

The formula to find the area of a triangle is $A=\frac{b\times h}{2}$

Replace A with 6. $6=\frac{b\times h}{2}$

Multiply both sides of $6=\frac{b\times h}{2}$ by 3.

$$3\times 6=\frac{3\times b\times h}{2}$$

$$18=\frac{3h\times b}{2}$$

Since 3h = 4b, we can replace 3h with 4b in the equation above.

$$18=\frac{4b\times b}{2}$$

$$18=\frac{4b^2}{2}$$

18 = $2b^2$

Divide both sides by 2

9 = b^2

Since 3^2 = 9, b = 3

Since the triangle is equilateral, perimeter = 3 + 3 + 3 = 9.

25.

The percent return rate of a growth fund, income fund, and money market are 10%, 7%, and 5% respectively. Suppose you have 3200 to invest and you want to put twice as much in the growth fund as in the money market to maximize your return. How should you invest to get a return of 250 dollars in 1 year?

Trick: Try to model the problem with equations.

Growth fund + income fund + money market = 3200

Growth fund = 2 x money market

10% of growth fund + 7% of income fund + 5% of money market = 250

Let x = growth fund, y = income fund, and z = money market.

x + y + z = 3200 equation 1

x = 2z equation 2

0.10x + 0.07y + 0.05z = 250 equation 3

We now have a system with 3 variables to solve.

Substitute 2z for x in equation 1 and equation 3

Equation 1 becomes $2z + y + z = 3200$ or $3z + y = 3200$

Equation 3 becomes 0.10 times $2z + 0.07y + 0.05z = 250$

Equation 3 becomes $0.2z + 0.07y + 0.05z = 250$ or $0.25z + 0.07y = 250$

Now we have two new equations

$3z + y = 3200$ equation 4

$0.25z + 0.07y = 250$ equation 5

Use equation 4 to solve for y

$3z + y = 3200$

$3z - 3z + y = 3200 - 3z$

$y = 3200 - 3z$

Replace y with $3200 - 3z$ in equation 5

$0.25z + 0.07(3200 - 3z) = 250$

$0.25z + 0.07 \times 3200 - 0.07 \times 3z = 250$

$0.25z + 224 - 0.21z = 250$

$0.25z - 0.21z + 224 = 250$

$0.04z + 224 = 250$

$0.04z + 224 - 224 = 250 - 224$

$0.04z = 26$

Divide both sides by 0.04

$$\frac{0.04}{0.04}z = \frac{26}{0.04}$$

$z = 650$

$x = 2z = 2 \text{ times } 650 = 1300$

$x + y + z = 3200$

$1300 + y + 650 = 3200$

$1950 + y = 3200$

$y = 1250$

26.

A shark was caught whose tail weighted 200 pounds. The head of the shark weighted as much as its tail plus half its body. Its body weighted as much as its head and tail. What is the weight of the shark?

Trick: Try to model the problem with equations.

Weight of Head + weight of body + weight of tail = weight of shark

Weight of head = weight of tail + half of weight of body

Weight of body = weight of head + weight of tail

Let H be the weight of the head

Let B be the weight of the body

Let T be the weight of the tail.

$H + B + T$ = weight of shark

$H = T + 1/2(B)$

$B = H + T$

Substitute 200 for T

$H + B + 200$ = weight of shark   equation 1

$H = 200 + 1/2(B)$ equation 2

$B = H + 200$ equation 3

Use equation 2 and equation 3 to solve for H and B.

$H = 200 + 1/2(B)$ equation 2

$B = H + 200$ equation 3

Multiply equation 2 by 2 and keep equation 3 the same.

$2H = 400 + B$

$B = H + 200$

Substitute $H + 200$ for B in $2H = 400 + B$

$2H = 400 + H + 200$

$2H = 600 + H$

$2H - H = 600$

H = 600

B = H + 200 = 600 + 200 = 800

Weight of shark = 600 + 800 + 200 = 1600.

27.

The square root of a number plus two is the same as the number. What is the number?

Trick: The square root of a number plus 2 is not written as $\sqrt{x+2}$

Write it as $\sqrt{x}$ + 2

Therefore, we need to solve $\sqrt{x}$ + 2 = x

Subtract 2 from both sides of the equations.

$\sqrt{x}$ + 2 - 2 = x - 2

$\sqrt{x}$ = x – 2

Raise both sides to the second power.

$$(\sqrt{x})^2 = (x-2)^2$$

$$x = x^2-4x+4$$

$$x - x = x^2-4x+4 \ -x$$

$$0 = x^2-5x+4$$

Factor $x^2 - 5x + 4$

$x^2 - 5x + 4$ = (x + a)(x + b)

Look for two numbers a and b whose product is 4 and the sum us -5

These two numbers are -4 and -1 since -1 times -4 = 4 and -4 + -1 = -5

(x + -4)(x + -1) = 0

x + - 4 = 0 and (x + -1) = 0

x = 4 and x = 1

Check $\sqrt{x}$ + 2 = x

Is $\sqrt{4}$ + 2 = 4 Is $\sqrt{1}$ + 2 = 1 ?

2 + 2 = 4 $1 + 2 \neq 1$

The number we are looking for is 4.

28.

Suppose you have a coupon worth 6 dollars off any item at a mall. You go to a store at the mall that offers a 20% discount. What do you need to do to save the most money?

Trick: Find two functions that model the cost.

Compute the cost by applying the coupon first and then the discount.

Compute the cost by applying the discount first and then the coupon.

Let f(x) = x – 6 be the cost with a coupon of 6 dollars

Let g(x) = 0.80x be the cost with a 20% discount.

Let g(f(x)) be the cost when the coupon is applied first.

Let f(g(x)) be the cost when the discount is applied first.

g(f(x)) = g(x – 6) = 0.80 (x – 6) = 0.80x – 0.48

f(g(x)) = f(0.80x) = 0.80x – 6

To save the most money, you need to tell the salesperson to apply the discount first.

29.

Suppose your grades on three math exams are 80, 93, and 91. What grade do you need on your next exam to have at least a 90 average on the four exams?

Trick: The average of should be at least 90. In other words, $average \geqslant 90$

Let x be the grade on the fourth exam.

$$\frac{80+93+91+x}{4} \geqslant 90$$

$$\frac{264+x}{4} \geqslant 90$$

Multiply both sides by 4

$$4 \times \frac{264 + x}{4} \geqslant 90 \times 4$$

$$264 + x \geqslant 360$$

$$264 - 264 + x \geqslant 360 - 264$$

$$x \geqslant 96$$

You should have at least a 96 on the fourth exam.

30.

Peter has a photograph that is 5 inches wide and 6 inches long. She enlarged each side by the same amount. By how much was the photograph enlarged if the new area is 182 square inches?

Trick:The width is by enlarged by x and the length was also enlarged by x

The width is now 5 + x and the length is 6 + x

Area = (5 + x)(6 + x) = 182

(5 + x)(6 + x) = 182

30 + 5x + 6x + x^2 = 182

30 + 11x + x^2 = 182

30 - 182 + 11x + x^2 = 182 - 182

x^2 + 11x – 152 = 0

$(x + a)(x + b) = 0$

To find a and b, look for two factors of -152 that will add up to 11

These two factors are a and b.

$-152 = -8 \times 19$ and $-8 + 19 = 11$

$(x + -8)(x + 19) = 0$

$x + -8 = 0$ and $(x + 19) = 0$

$x + - 8 = 0$

$x = 8$ since $8 + -8 = 0$

$x + 19 = 0$

$x = -19$ since $-19 + 19 = 0$. How $x = -19$ is not a solution since the dimensions cannot be negative.

$X = 8$ is the amount the photograph was enlarged by.

31.

The cost to produce a book is 1200 to get started plus 9 dollars per book. The book sells for 15 dollars each. How many books must be sold to make a profit?

Trick: To make a profit, revenue must be greater than cost.

Revenue > cost

Let y be the number of books sold.

Revenue $= 15 \times y = 15y$

Cost = 1200 + 9y

Revenue > cost when 15y > 1200 + 9y

Solve the inequality

15y > 1200 + 9y

15y – 9y > 1200

6y > 1200

$\frac{6}{6}$ y > $\frac{1200}{6}$

y > 200

To make a profit, more than 200 books should be sold. At the very least 201 books must be sold.

32.

Store A sells CDs for 2 dollars each if you pay a one-time fee of 104 dollars. Store B offers 12 free CDs and charges 10 dollars for each additional CD. How many CD must you buy so it will cost the same under both plans?

Trick: Cost at store A = Cost at store B.

Let x be the number of CD you buy. Since store B offers 12 free CDs, you are really paying for x – 12 CDs

Cost at store A = 104 + 2x

Cost at store B = (x - 12)10

Cost at store A = Cost at store B when 104 + 2x = (x – 12)10

$104 + 2x = (x - 12)10$

$104 + 2x = 10x - 120$

$104 + 120 + 2x = 10x - 120 + 120$

$224 + 2x = 10x$

$10x = 2x + 224$

$10x - 2x = 2x - 2x + 224$

$8x = 224$

Since 8 times 28 = 224, x = 28.

For the cost to be the same at both both store, you must buy 28 CDs.

33.

When 4 is added to two numbers, the ratio is 5:6. When 4 is subtracted from the two numbers, the ratio is 1:2. Find the two numbers.

Trick: Let x be the first number and let y be the second number. Add 4 to both numbers and set up a proportion. Then, subtract 4 from both numbers and set up another proportion.

Add 4 to both numbers. We get x + 4 and y + 4

The first proportion is $\frac{x+4}{y+4} = \frac{5}{6}$

Subtract 4 from both numbers. We get x - 4 and y – 4

The second proportion is $\frac{x-4}{y-4} = \frac{1}{2}$

Using $\frac{x+4}{y+4} = \frac{5}{6}$, cross multiplying

6 (x + 4) = 5(y + 4)

6x + 24 = 5y + 20

6x – 5y = 20 – 24

6x – 5y = -4

Using $\frac{x-4}{y-4} = \frac{1}{2}$, cross multiply.

2(x – 4) = y – 4

2x – 8 = y – 4

2x – y = -4 + 8

2x – y = 4

We then have the following two equations.

6x – 5y = -4 equation 1

2x – y = 4 equation 2

Multiply equation 2 by -5

6x – 5y = -4

$-10x + 5y = -20$

Add the two equations

$-4x = -24$

Since -4 times 6 = -24, x = 6

Using equation 2, 2x6 – y = 4

12 – y = 4

Since 12 – 4 = 8, y = 8

The two numbers are 6 and 8.

The ratio of x to y is 6:8

34.

A store owner wants to sell 200 pounds of pistachios and walnuts mixed together. Walnuts cost 4 dollars per pound and pistachio cost 6 dollars per pounds. How many pounds of each type of nuts should be mixed if the store owner will charge 5 dollars for the mixture?

Trick: cost of pistachios + cost of walnuts = cost of mixture (equation 1)

Cost of mixture = 200 x 5 = 1000 (equation 2)

Cost of pistachios = number of pounds x cost per pound. (equation 3)

Cost of walnuts = number of pounds x cost per pound. (equation 4)

Let y be the number of pounds used for pistachio, then 200 – y

is the number of pounds used for walnuts.

Equation 3 becomes   cost of pistachios = 6 times y = 6y

Equation 4 becomes   Cost of walnuts = 4 times (200 – y) = 800 – 4y

Replacing all known values in equation 1, we get:

6y + 800 – 4y = 1000

2y + 800 = 1000

2y + 800 – 800 = 1000 – 800

2y = 200

y = 100 since 2 times 100 = 200. Use 100 pounds of pistachios.

200 – y = 200 – 100 = 100 is the number of pounds for walnuts.

35.

A cereal box manufacturer makes 36-ounce boxes of cereal. In a perfect world,the box will be 36-ounce every time is made. However, since the world is not perfect, they allow a difference of 0.06 ounce. Find the range of acceptable size for the cereal box.

Trick: Problems like this are modeled with an absolute value inequality.

Let x be the size of the cereal box

$$|x-36| \leq 0.06$$

This will give us two equations.

$x-36\leq 0.06$ and $-(x-36)\leq 0.06$

Let us solve $x-36\leq 0.06$

$x-36\leq 0.06$

$x-36+36\leq 0.06+36$

$x\leq 0.06+36$

$x\leq 36.06$

Let us solve $-(x-36)\leq 0.06$

$-(x-36)\leq 0.06$

Multiply both sides by -1

-1 times $-(x-36)\leq 0.06$ times -1

$(x-36)\geq -0.06$ (Notice how we change the direction of the inequality)

$x-36+36\geq -0.06+36$

$x-36+36\geq -0.06+36$

$x\geq 35.94$

$x\geq 35.94$

$35.94 \leq x$

Let us put these two equations together $35.94 \leq x$ and $x \leq 36.06$

We get $35.94 \leq x \leq 36.06$

The cereal box can be at least 35.94 and at the most 36.06

36.

A man weighing 600 kg has been losing 3.12% of his weight each month with some heavy exercises and eating the right food. What will the man weigh after 20 months?

Trick: This problem is a typical example of exponential decay, so model the situation with an exponential decay equation.

The equation is y = a x b^x

y = final amount. In our example, it will be the final weight.

a = starting amount. In our example, it will be the initial weight.

b = decay factor. Another trick is to find the decay factor.

If I decrease 100 by 5%, I get 100 – 5 = 95. I can get 95 if I do 95% of 100 or 0.95 times 100 = 95. Therefore, to decrease a number by 5%, just times the number by 95% or 0.95.

In our problem, we are decreasing by 3.12%. It is the same as 100% - 3.12% = 96.88%. The decay factor is then 0.9688.

x = time elapsed. In our problem, it will be 20 months.

Now, we are ready to use the formula.

y = a x b^x

y = 600 x 0.9688^{20}

y = 600 x 0.5304

y = 318.24

After 20 months, the man will weigh about 318 pounds.

37.

An object is thrown into the air at a height of 60 feet. After 1 second and 2 second, the object is 88 feet and 84 feet in the air respectively. What is the initial speed of the object?

Trick: Know the formula that model the height of an object thrown in the air at a certain height. This is a quadratic equation as shown below.

H = f(t) = a t^2 + bt + c

b is the initial speed c is the initial height.

When the object is 60 feet in the air, it has not been thrown yet.

Therefore, t = 0 at a height of 60 feet.

f(0) = a 0^2 + b times + c

f(0) = 0 + 0 + c

f(0) = c

Since f(0) = H = 60, c = 60

f(t) = H = a t^2 + b x 1 + 60

f(1) = a x 1^2 + b + 60

88 = a + b + 60

88 – 60 = a + b

a + b = 28   equation 1

f(2) = a x 2^2 + b x 2 + 60

84 = 4a + 2b + 60

84 – 60 = 4a + 2b

4a + 2b = 24   equation 2

We can use equation 1 and equation 2 to solve for a and b.

a + b = 28

4a + 2b = 24

Multiply equation by -4 and keep equation 2 the same.

-4a + -4b = -112

4a + 2b = 24

Add the two equations

-4a + 4a + -4b + 2b = -112 + 24

0 + -2b = -88

-2b = -88

Since -2 times 44 = -88, b = 44

Therefore, the initial speed of the object is 44 feet per second.

Since a + b = 28, a + 44 = 28. a = -16 since -16 + 44 = 28.

f(t) = -16 t^2 + 44t + 60

Generally speaking, a is always equal to -16, so next time there is no need to look for it.

38.

A transit is 200 feet from the base of a building. There is man standing on top of the building. The angles of elevation from the top and bottom of the man are 45 degrees and 44 degrees. What is the height of the man?

Trick: recognize that this is a problem involving trigonometry. Try to understand the problem and make a drawing.

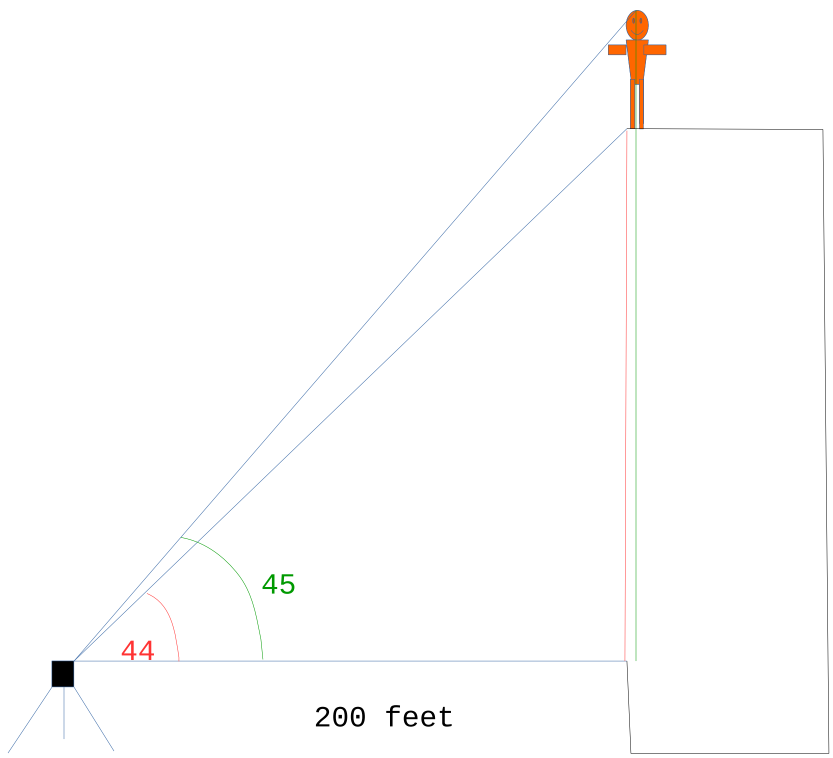
What is a transit? It is a tool used to measure angle.

What is an angle of elevation? It is the formed between the horizontal and the line formed from the object to the transit.

In this problem, there are two angles of elevations.

The first one is the angle between the horizontal and the line formed from the top of the man (or the top of the man’s head) and the transit.

The second one is the angle between the horizontal and the line formed from the bottom of the man (or the bottom of the man’s feet) and the transit.



Let x be the green line and let y be the red line.

Red line – green line = height of man.

Let us find x

tan (45 degrees) = $\frac{x}{200}$

Multiply both sides by 200

200 x tan(45 degrees) = $\frac{x}{200}$ x 200

200 x tan(45 degrees) = x

Since tan(45 degrees) = 1,   x = 200 x 1 = 200 feet

Let us find y

tan (44 degrees) = $\frac{y}{200}$

Multiply both sides by 200

200 x tan(44 degrees) = $\frac{y}{200}$ x 200

200 x tan(44 degrees) = y

Since tan(44 degrees) = 0.965, y = 200 times 0.965 = 193

The height of the man is x – y = 200 – 193 = 7 feet.

39.

A lemonade consists of 6% of lemon juice and a strawberry juice consists of 15% pure fruit juice. How much of each kind should be mixed together to get a 4 Liters of a 10% concentration of fruit juice?

Trick:

Number of liter of water containing 6% of lemon juice + number of liter of water containing 15% of strawberry juice = number of liter containing 10% concentration of fruit juice.

Concentration of lemon + concentration of strawberry juice = concentration of mixture.

Let x be the number of liter of water that contains 6% of lemon juice.

Let y be the number of liter of water that contains 15% of strawberry juice.

$x + y = 4$

6% of x + 15% of y = 10% of 4

$x + y = 4$ equation 1

$0.06x + 0.15y = 0.4$ equation 2

Solve for x in equation 1

$x + y = 4$

$x + y - y = 4 - y$

$x = 4 - y$

Replace 4 – y in equation 2

$0.06(4 - y) + 0.15y = 0.4$

0.06 times 4 – 0.06 times y + 0.15y = 0.4

$0.24 + -0.06y + 0.15y = 0.4$

$0.24 + 0.09y = 0.4$

$0.24 - 0.24 + 0.09y = 0.4 - 0.24$

$0.09y = 0.16$

$y = \frac{0.16}{0.09}$

$y = 1.78$

$x = 4 - 1.77$

$x = 2.22$

Therefore, we need to mix 1.77 L of water containing 6% lemon juice with 2.22 L of water containing 15% of strawberry juice.

40.

Ellen can wash her car in 60 minutes. Her older sister Sarah can do the same job in 45 minutes. How long will it take if they wash the car together?

Trick: Fraction of job Ellen can do in 1 minute + fraction of job Sarah can do in 1 minute = fraction of job they can complete in 1 minute.

Let n be time it takes both sisters to complete the work together.

$$\frac{1}{60} + \frac{1}{45} = \frac{1}{n}$$

Multiply everything by 180n

$$180n \left(\frac{1}{60} + \frac{1}{45} \right) = 180n \frac{1}{n}$$

$$\frac{180n}{60} + \frac{180n}{45} = \frac{180n}{n}$$

$$3n + 4n = 180$$

$$7n = 180$$

$$n = 25.71$$

0.71 * 60 second = 42.6 seconds

They can do the job in 25 minutes and 42.6 seconds.

41.

A plane flies 500 mi/h. The plane can travel 1100 miles with the wind in the same amount of time as it travels 900 miles against the wind. What is the speed of the wind?

Trick: Use the equation t = $\frac{distance}{speed}$

There are two situations. Let w be the speed of the wind.

Situation #1:

The plane travels with the wind. In this case, the speed is v + w

t = $\frac{1100}{500+w}$

Situation #2:

The plane travels against the wind. In this case, speed is v – w

t = $\frac{900}{500-w}$

The time it takes to travel these two distance is the same

Therefore,

$$\frac{1100}{500+w} = \frac{900}{500-w}$$

Cross multiplying

$1100(500 - w) = 900(500 + w)$

$550000 - 1100w = 450000 + 900w$

$550000 - 450000 - 1100w = 900w$

$100000 - 1100w = 900w$

$100000 - 1100w + 1100w = 900w + 1100w$

$100000 = 2000w$

$w = \dfrac{100000}{2000}$

$w = 50$

The speed of the wind is 50 mi/h

42.

A company produces boxes that are 5 feet long, 4 feet wide, and 3 feet high. The company wants to increase each dimension by the same amount so that the new volume is twice as big. How much is the increase in dimension?

Trick:

Let x be the increase, then the new dimensions are 5 + x, 4 + x, and 3 + x

New volume = 2 x (5 x 4 x 3) = 2 x 60 = 120

120 = (5 + x)(4 + x)(3 + x)

120 = [(5 + x)(4 + x)](3 + x)

120 = [5 times 4 + 5 times x + x times 4 + x times x](3 + x)

120 = [20 + 5x + 4x + x^2](3 + x)

120 = [20 + 9x + x^2](3 + x)

120 = (20 times 3 + 20 times x + 9x times 3 + 9x times x + x^2 times 3 + x^2 times x

120 = 60 + 20x + 27x + 9 x^2 + 3 x^2 + x^3

120 = 60 + 47x + 12 x^2 + x^3

120 – 120 = 60 – 120 + 47x + 12 x^2 + x^3

47x + 12 x^2 + x^3 - 60 = 0

x^3 + 12 x^2 + 47x - 60 = 0

We need to find a number x that will make this equation equal to 0.

We need to use the rational root theorem.

By the rational roots theorem, all possible rational roots have the form

$$\frac{factor\ of -60}{factor\ of\ 1}$$

All possible factors of -60 are ± 1 and ± 60 , ± 2 and ± 30 ,

± 3 and ± 20 , ± 4 and ± 15 , ± 5 and ± 12 , and ± 6 and ± 10

All possible factors of 1 are ± 1

Let ignore the sign of the answer for now.

The division will give the following numbers

1, 2, 3, 4, 5, 6, 12, 15, 20, 30

When a positive/negative is divided by another positive/negative, the answer can be either positive or negative. Therefore, all we need to do is to put positive or negative next 1, 2, 3, 4, 5, 6, 12, 15, 20, 30

all possible rational roots are ± 1 , ± 2 , ± 3 , ± 4 , ± 5 , ± 6 ,

± 12 , ± 15 , ± 20 , and ± 30

There are a lot of numbers to try. Always start with the smallest numbers first.

Let us try 1

x^3 + 12 x^2 + 47x - 60 = 1^3 + 12 1^2 + 47 times 1 - 60 = 1 + 12 + 47 – 60

= 13 + 47 – 60

$= 60 – 60$

$= 0$

x = 1 works!

This means that (x – 1) is a factor of $x^3 + 12x^2 + 47x - 60$

Divide $x^3 + 12x^2 + 47x - 60$ by (x – 1)

Use synthetic division

Write x - 1 $\overline{|\ x^3 + 12x^2 + 47x - 60}$

as 1 | 1 12 47 -60

$$\begin{array}{r|rrrr} 1 & 1 & 12 & 47 & -60 \\ & & 1 & & \\ \hline & 1 & 13 & & \end{array}$$

(1 times 1 = 1. Add 1 to 12)

$$\begin{array}{r|rrrr} 1 & 1 & 12 & 47 & -60 \\ & & 1 & 13 & \\ \hline & 1 & 13 & 60 & \end{array}$$

(1 times 13 = 13. Add 13 to 47)

1	|	1	12	47	-60
			1	13	60
		1	13	60	0

(1 times 60 = 60. Add 60 to -60)

The answer is x^2 + 13x + 60

Let us now factor x^2 + 13x + 60 by looking for factors of 60 that add up to 13.

There are no such factors.

$x^3 + 12x^2 + 47x - 60 = (x - 1)(x^2 + 13x + 60)$

The only real number that solve $x^3 + 12x^2 + 47x - 60$ is x = 1

The new dimensions are 5 + x, 4 + x, and 3 + x or 6, 5, 4

Indeed, 6 times 5 times 4 = 30 times 4 = 120

43.

James invested half of his money in land, a tenth in stock, and a twentieth in saving bonds. Then, he put the remaining 21000 in a CD. How much money did James saved or invested?

Trick: Money James had minus money invested in land minus money invested in stock minus money invested in saving bonds equal money he put in a CD.

Let x be the total money he saved or invested

$$x - \frac{1}{2} x - \frac{1}{10} x - \frac{1}{20} x = 21000$$

$$\frac{1}{1} x - \frac{1}{2} x - \frac{1}{10} x - \frac{1}{20} x = 21000$$

$$\frac{20}{20} \text{ times } \frac{1}{1} x - \frac{10}{10} \text{ times } \frac{1}{2} x - \frac{2}{2} \text{ times } \frac{1}{10} x - \frac{1}{20} x = 21000$$

$$\frac{20}{20} x - \frac{10}{20} x - \frac{2}{20} x - \frac{1}{20} x = 21000$$

$$\frac{20-10-2-1}{20} x = 21000$$

$$\frac{7}{20} x = 21000$$

$$\frac{20}{7} \text{ times } \frac{7}{20} x = 21000 \text{ times } \frac{20}{7}$$

$$x = \frac{21000}{7} \text{ times } 20$$

$$x = 3000 \text{ times } 20$$

$$x = 60000$$

44.

The motherboards for a desktop computer can be manufactured for 50 dollars each. The development cost is 250000. The first 20 motherboard are samples and will not be sold. How many salable motherboards will yield an average cost of 6325 dollars?

Trick: Be able to write the average cost correctly. Understand the meaning of the word salable. Salable = fit to be sold

Average cost = total cost **divided** by number of **salable** motherboards

Let x be the number of motherboard produced. Then x – 20 = number of salable motherboards.

Total cost = 50x + 250000

Average cost = $\frac{50x+250000}{x-20}$

Replace average cost with 6325

6325 = $\frac{50x+250000}{x-20}$

Multiply both sides by x – 20

(x – 20)6325 = 50x + 250000

$6325x - 126500 = 50x + 250000$

$6325x - 126500 + 126500 = 50x + 250000 + 126500$

$6325x = 50x + 376500$

$6325x - 50x = 50x - 50x + 376500$

$6325x - 50x = 376500$

$6275x = 376500$

$$x = \frac{376500}{6275}$$

$x = 60$

45.

How much of a 70% orange juice drink must be mixed with 44 gallons of a 20% orange juice drink to obtain a mixture that is 50% orange juice?

Trick:

70% concentration of orange juice + 20% concentration of orange juice = 50% concentration of orange juice.

Let n be the number of gallons of 70% orange juice.

Let 44 be the number of gallons of 20% orange juice.

70% of n + 20% of 44 = 50% of (n + 44)

0.70n + 8.8 = 0.50(n + 44)

0.70n + 8.8 = 0.50n + 0.50 x 44

0.70n + 8.8 = 0.50n + 22

0.70n + 8.8 – 8.8 = 0.50n + 22 – 8.8

0.70n = 0.50n + 13.2

0.70n – 0.50n = 0.50n – 0.50n + 13.2

0.20n = 13.2

$$n = \frac{13.2}{0.20}$$

n = 66

46.

A company sells nuts in bulk quantities. When bought in bulk, peanuts sell for $1.20 per pound, almonds for $ 2.20 per pound, and cashews for $3.20 per pound. Suppose a specialty shop wants a mixture of 280 pounds that will cost $2.59 per pound. Find the number of pounds of each type of nut if the sum of the number of pounds of almonds and cashews is three times the number of pounds of peanuts. Round your answers to the nearest pound.

Trick:

number of pound of peanuts + number of pound of almonds + number of pound of cashews = 280

cost of peanuts + cost of almonds + cost of cashews = 2.59 x 280 = 725.2

number of pounds of almonds and cashews = 3 times the number of pounds of peanuts.

Let x be the number of pound of peanuts

Let y be the number of pound of almonds

Let z be the number of pound of cashews.

$x + y + z = 280$ equation 1

$1.20x + 2.20y + 3.20z = 725.2$ equation 2

$y + z = 3x$ equation 3

Replace y + z with 3x in equation 1

$x + 3x = 280$

$4x = 280$

x = 70 since 70 times 4 = 280

Replace x with 70 in equation 2

$1.20 \text{ times } 70 + 2.20y + 3.20z = 725.2$

$84 + 2.20y + 3.20z = 725.2$

$84 - 84 + 2.20y + 3.20z = 725.2 - 84$

$2.20y + 3.20z = 641.2$ equation 4

Using equation 3, $y + z = 3 \text{ times } 70 = 210$

$2.20y + 3.20z = 641.2$ equation 4

$y + z = 210$ equation 3

Using equation 3, solve for y

$y + z = 210$

$y + z - z = 210 - z$

$y = 210 - z$

Replace the value of y in equation 4

$2.20(210 - z) + 3.20z = 641.2$

$2.20 \text{ times } 210 - 2.20z + 3.20z = 641.2$

$462 - 2.20z + 3.20z = 641.2$

$462 + 1z = 641.2$

$462 - 462 + 1z = 641.2 - 462$

$1z = 179.2$

$z = 179.2$

$y = 210 - 179.2 = 30.8$

Therefore, mix 70 pounds of peanuts, 30.8 pounds of almonds, and 179.2 pounds of cashews together.

47.

A Basketball player has successfully made 36 of his last 48 free throws. Find the number of consecutive free throws the player needs to increase his success rate to 80%.

Trick: Know how to model the success rate.

For a success rate of 36 out of 48, it is $\frac{36}{48} = 75\%$

Therefore, success rate = number of successful throws divided by number of free throws.

Suppose she makes x more consecutive free throws.

Success rate = $\frac{36+x}{48+x}$

To raise his success rate to 80%, $\frac{36+x}{48+x}$ must equal to 80%

$\frac{36+x}{48+x}$ = 0.80

Multiply both sides by 48 + x

(48 + x) ($\frac{36+x}{48+x}$) = 0.80(48 + x)

36 + x = 0.80 (48 + x)

36 + x = 0.80 times 48 + 0.80x

36 + x = 38.4 + 0.80x

36 – 36 + x = 38.4 – 36 + 0.80x

x = 2.4 + 0.80x

x – 0.80x = 2.4 + 0.80x – 0.80x

0.20x = 2.4

x = $\frac{2.4}{0.20}$

x = 12

He will need 12 throws to reach a success rate of 80%

48.

John can wash cars 3 times as fast as his son Erick. Working together, they need to wash 30 cars in 6 hours. How many hours will it takes each of them working alone?

Trick: John's rate + Erick's rate = combined rate

Let x be the time it takes John. John's rate = $\frac{30}{x}$

Let 3x be the time it takes Erick. Erick's rate = $\frac{30}{3x}$

Let 6 hours be the time they must finish Combined rate = $\frac{30}{6}$ = 5

$$\frac{30}{x} + \frac{30}{3x} = 5$$

$$\frac{30}{x} + \frac{3 \times 10}{3x} = 5$$

$\frac{30}{x} + \frac{10}{x} = 5$

$\frac{30+10}{x} = 5$

$\frac{40}{x} = 5$

Multiply both sides by x

x times $\frac{40}{x}$ = 5 times x

40 = 5x

Since 5 times 8 = 40, x = 8

Erick’s time = 3 times 8 = 24

49.

In a college, about 36% of student are under 20 years old and 15% are over 40 years old. What is the probability that a student chosen at random is under 20 years old or over 40 years?

Trick: A student cannot be 20 years old and over 40 years at the same.

Therefore, use the formula for mutually exclusive events.

P(under 20 or over 40) = P(under 20) + P(over 40)

P(under 20 or over 40) = 0.36 + 0.15

P(under 20 or over 40) = 0.51

50.

Twice a number plus the square root of the number is twelve minus the square root of the number.

Trick:

Let x be the number. Twice the number = 2x

Square root of the number = $\sqrt{x}$

Replace is with =

$2x + \sqrt{x} = 12 - \sqrt{x}$

$2x - 2x + \sqrt{x} = 12 - 2x + \sqrt{x}$

$\sqrt{x} = 12 - 2x - \sqrt{x}$

$\sqrt{x} + \sqrt{x} = 12 - 2x - \sqrt{x} + \sqrt{x}$

$2\sqrt{x} = 12 - 2x$

Divide everything by 2

$\sqrt{x}$ = 6 – x

Raise both sides to the second power.

$(\sqrt{x})^2$ = $(6-x)^2$

x = 36 – 12x + x^2

x – x = 36 – 12x – x + x^2

0 = 36 – 13x + x^2

0 = (x – 4)(x - 9)

x = 4 and x = 9

Check

2 times 4 + $\sqrt{4}$ = 12 - $\sqrt{4}$

8 + 2 = 12 – 2

10 = 10

2 times 9 + $\sqrt{9}$ = 12 - $\sqrt{9}$

18 + 3 is not equal to 12 - 3

The answer is x = 4

51.

The light intensity, I , of a light bulb varies inversely as the square of the distance from the bulb. A a distance of 3 meters from the bulb, I = 1.5 W/m^2 . What is the light intensity at a distance of 2 meters from the bulb?

Trick: Understand the meaning of varies inversely. The word inverse indicates taking the inverse. Therefore, we need to take the inverse of something. We will take the inverse of the square of the distance as stated in the problem

The square of the distance is d^2

So far, we have I = $\frac{1}{d^2}$

This is not all though. The word varies inversely also means that

I and $\frac{1}{d^2}$ are in proportion.

In other words, $\frac{I}{\frac{1}{d^2}}=k$

Multiply both sides of the equation by $\frac{1}{d^2}$

I = k $\frac{1}{d^2}$

Now, replace I with 1.5 and d with 3 to solve for k

1.5 = k $\frac{1}{3^2}$

1.5 = k $\frac{1}{9}$

1.5 = $\frac{k}{9}$

Multiply both sides by 9

1.5 x 9 = k

k = 13.5

I = 13.5 $\frac{1}{d^2}$

Now, find I when d = 2

I = 13.5 $\frac{1}{2^2}$

I = 13.5 $\frac{1}{4}$

I = $\frac{13.5}{4}$

I = 3.375 W/m^2

52.

The lengths of two sides of a triangle are 2 and 6. find the range of values for the possible lengths of the third side.

Trick: The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Let x be the length of the third side.

Then,using the trick above, we get 3 inequalities

$2 + x > 6$, $2 + 6 > x$, and $6 + x > 2$

$2 + x > 6$

$2 - 2 + x > 6 - 2$

$0 + x > 4$

$x > 4$

$2 + 6 > x$

$8 > x$

$x < 8$

$6 + x > 2$

$6 - 6 + x > 2 - 6$

$x > -4$

The answer $x > -4$ does not make sense. Since one of the side is equal to 2 x must be at least 5 or bigger than 4 so that the sum of these two sides can be bigger than 6.

Therefore, $x > 4$ and $x < 8$

we can also write $4 < x < 8$

53.

Find three consecutive integers such that one half of their sum is between 15 and 21.

Trick: Know how to write the first even integer. Let 2n be the first even integer, Then 2n + 2 is the second, and 2n + 4 is the third.

The sum is $2n + 2n + 2 + 2n + 4 = 6n + 6$

Half of their sum is $\frac{6n+6}{2} = 3n + 3$

$3n + 3$ is between 15 and 21

$15 < 3n + 3 < 21$

$15 - 3 < 3n + 3 - 3 < 21 - 3$

$12 < 3n < 18$

$12/3 < 3n/3 < 18/3$

$4 < n < 6$

$n = 5$

The first even integer is 2n = 2 times 5 = 10

The second even integer is 12 and the third is 14

Indeed, $\frac{10+12+14}{2} = \frac{36}{2} = 18$

and 18 is between 15 and 21

54.

After you open a book, you notice that the product of the two page numbers on the facing pages is 650. What are the two page numbers?

Trick: One of the page will show an even number and the other will be an odd number.

Let 2n be one of the two facing pages. Then, the other page is 2n + 1

$2n(2n + 1) = 650$

$4n^2 + 2n = 650$

$4n^2 + 2n - 650 = 0$

Divide everything by 2

$2n^2$ + n – 325 = 0

(2n + blank)(n + blank) = 0

Notice that -25 x 13 = -325

Notice also that 2n x 13 + n x -25 = 26n – 25n = n

We get (2n + -25)(n + 13) = 0

2n + - 25 = 0

2n + - 25 + 25 = 0 + 25

2n + 0 = 25

2n = 25

n = 25/2 = 12.5

n + 13 = 0

n + 13 – 13 = 0 – 13

n = -13

Since a page cannot be negative n = - 13 is not a solution.

N = 12.5

The first page is 2n or 2 x 12.5 = 25

The second page is 26.

The two facing pages are 25 and 26

55.

Suppose you start with a number. You multiply the number by 3, add 7, divide by ½, subtract 5, and then divide by 12. The result is 5. What number did you start with?

Trick: Just let x be the number and just do the math in the problem. Be careful when dividing by ½.

Multiply the number by 3 : x times 3 = 3x

Add 7: 3x + 7

Divide by ½: $\frac{3x+7}{\frac{1}{2}}$ = (3x + 7) times 2 = 2(3x + 7)

Subtract 5: 2(3x + 7) – 5

Divide by 12: $\frac{2(3x+7)-5}{12}$

Result is 5: $\frac{2(3x+7)-5}{12}$ = 5

Now solve the equation $\frac{2(3x+7)-5}{12}$ = 5

Multiply both sides by 12

12 times $\frac{2(3x+7)-5}{12}$ = 5 times 12

2(3x + 7) – 5 = 60

2(3x + 7) – 5 + 5 = 60 + 5

2(3x + 7) = 65

6x + 14 = 65

6x + 14 – 14 = 65 – 14

6x = 51

x = 8.5

56.

You have 156 feet of fencing to enclose a rectangular garden. You want the garden to be 5 times as long as it is wide. Find the dimensions of the garden.

Trick: Perimeter = 2 x length + 2 x width

Length = 5 x width

Carefully replace everything you know into the formula.

156 = 2 x 5 x width + 2 x width

156 = 10 width + 2 width

156 = 12 width

width = $\frac{156}{12}$ = 13

Length = 5 x width = 5 times 13 = 65

57.

The amount of water a dripping faucet wastes water varies directly with the amount of time the faucet drips. If the faucet drips 2 cups of water every 6 minutes, find out how long it will take the faucet to drip 10.6465 liters of water.

Trick: Understand the meaning of varies directly. If x varies directly with y, then x / y = k

Make sure you convert 10.6465 liters to cups before doing any math computation.

1 liter = 4.22675 cups

so 10.6465 liters = 10.6465 x 4.22675 cups = 45 cups

Let w = number of cups of water wasted

Let t = time in minutes the faucet drips

$$\frac{w}{t} = k$$

Find k. $k = \frac{w}{t} = \frac{2}{6} = \frac{1}{3}$

$$\frac{w}{t} = \frac{1}{3}$$

Find t when w = 45 cups

$$\frac{45}{t} = \frac{1}{3}$$

Cross multiply

t times 1 = 45 times 3

t = 135

58.

A washer costs 25% more than a dryer. If the store clerk gave a 10% discount for the dryer and a 20% discount for the washer, how much is the washer before the discount if you paid 1900 dollars.

Trick: Cost of dryer after discount + cost of washer after discount = 1900

Let x be the cost of the dryer before discount.

The cost of the washer is 25% more than the cost of the dryer

25% of x = 0.25x

x + 0.25x = 1.25x

The cost of the washer is 1.25x

Now, let us apply the discounts

10% of x = 0.10x

x – 0.10x = 0.90x. So the cost of the dryer after discount is 0.90x

20% of 1.25x = 0.20 times 1.25x = 0.25x

1.25x – 0.25x = 1x. So the cost of the washer after discount is 1x

Cost of dryer after discount + cost of washer after discount = 1900

$0.90x + 1x = 1900$

$1.90x = 1900$

$x = \frac{1900}{1.90} = 1000$

The cost of the dryer before discount is 1000

The cost of the washer before discount is 1.25x = 1.25 times 1000 = 1250

59.

Baking a tray of blueberry muffins takes 4 cups of milk and 3 cups of wheat flour. A tray of pumpkin muffins takes 2 cups of milk and 3 cups of wheat flour. A baker has 16 cups of milk and 15 cups of wheat flour. You make 3 dollars profit per tray of blueberry muffins and 2 dollars profit per tray of pumpkin muffins. How many trays of each type of muffins should you make to maximize profit?

Trick: Use vertex principle Of linear programming. This principle says that if there is a maximum or minimum value of the linear objective function, it occurs at one or more vertices of the feasible region.

Use a table to model the problem.

Let x be the number of trays of blueberry muffins.

Let y be the number of trays of pumpkin muffins.

	Blueberry muffins	Pumpkin muffins	Total
Number of trays	x	y	x + y
Number of cups of flour	3x	3y	15
Number of cups of milk	4x	2y	16
Profit	3x	2y	3x + 2y

We have three linear inequalities to graph

$3x+3y\leq 15$

$4x+2y\leq 16$

$x\geqslant 0$, $y\geqslant 0$

The profit equation = P = 3x + 2y

We can simplify these inequalities to this.

$x+y\leq 5$

$2x+y\leq 8$

$x\geqslant 0$, $y\geqslant 0$

Now try to graph the equality

x + y = 5 and 2x + y = 8

We can easily graph these two equations by looking for x-intercepts and y-intercepts.

For x + y = 5, if x = 0, then y = 5. The y-intercept for this equation is (0, 5)

For x + y = 5, if y = 0, then x = 5. The x-intercept for this equation is (5, 0)

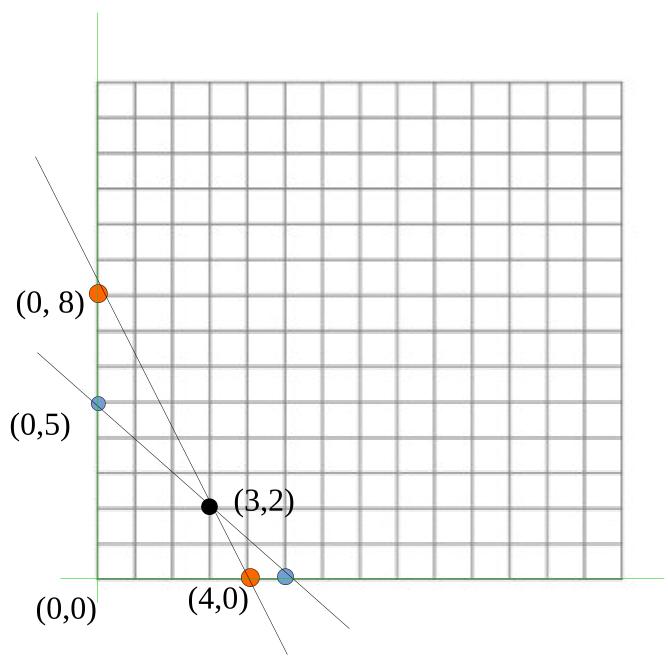
For $2x + y = 8$, if $x = 0$, then $y = 8$. The y-intercept for this equation is (0, 8)

For $2x + y = 8$, if $y = 0$, then $2x = 8$. $x = 4$.The x-intercept for this equation is (4, 0)

Use (0, 5) and (5, 0) to graph $x + y = 5$

Use (0, 8) and (4, 0) to graph $2x + y = 8$

The graphs are shown below.



We can find the point of intersection by solving the system

$x + y = 5$ and $2x + y = 8$

Using $x + y = 5$, $y = 5 – x$

Replace y = 5 – x in 2x + y = 8

2x + 5 – x = 8

x = 8 – 5

x = 3

x + y = 5

3 + y = 5

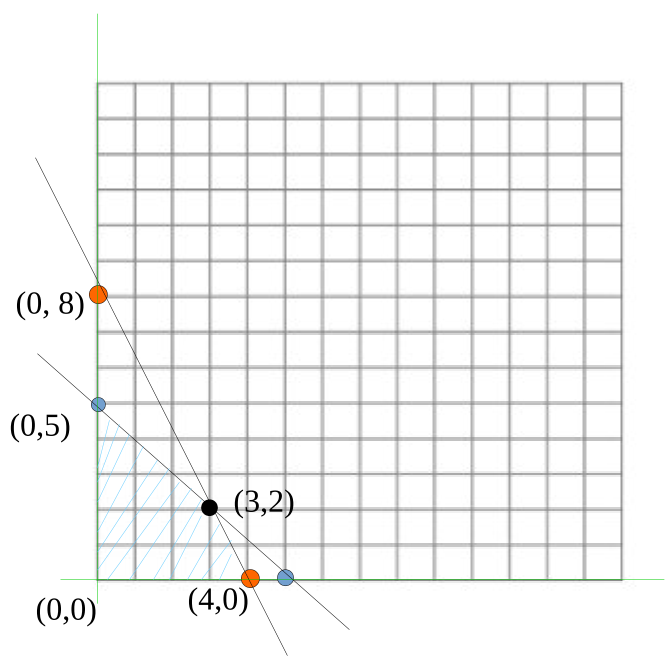
y = 5 – 3 = 2

The point of intersection is (3, 2)

The area of interest is shown with blue lines.

According to the vertex principle of linear programming, the maximum is found at the vertices of the feasible region (shown in blue below)

The vertices are (0, 0), (4, 0), (3, 2), and (0, 8)



P = 3x + 2y

Let us try these vertices and see which one yield the highest value.

With (0, 0), we get P = 3 x 0 + 2 x 0 = 0

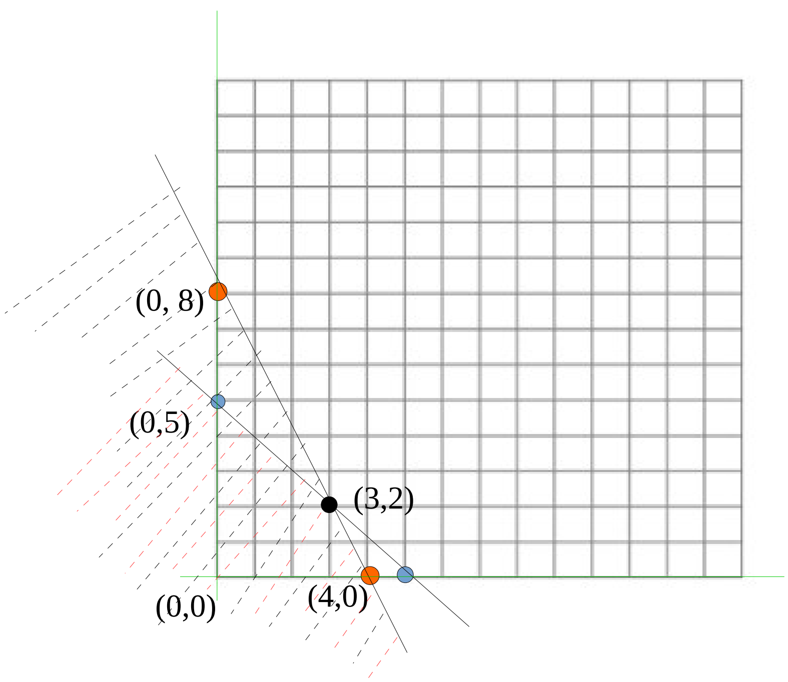
With (4, 0), we get P = 3 x 4 + 2 x 0 = 12 + 0 = 12

With (0, 5), we get P = 3 x 0 + 2 x 5 = 10

With (3, 2), we get P = 3 x 3 + 2 x 2 = 9 + 4 = 13

Therefore, make 3 trays of blueberry muffins and 2 trays of pumpkin muffins to maximize profit.

How did we find the blue area?



$2x+y\leq 8$ Or $y\leq 8-2x$

This means the area below 8 – 2x or everything below the line with points (0, 8) and (4, 0). We show the shade with black lines.

$x+y\leq 5$ Or $y\leq 5-x$

This means the area below 5 – x or everything below the line with points (0, 5) and (5, 0). We show the shade with red lines.

The area where you see black and red at the same time is what you need and it has to be within the boundary of green lines since $x\geqslant 0$ and $y\geqslant 0$

60.

A company found that -2p + 1000 models the number of TVs sold per month where p can be set as low as 200 or as high as 300. How can the company maximize the revenue?

Trick: Revenue = price per unit x number of units sold

Let p be the price per unit

Revenue = p (-2p + 1000)

Revenue = $-2p^2$ + 1000p

a = -2. Since a or the coefficient of $-2p^2$ is negative, the graph

$-2p^2$ + 1000p opens downward.

Therefore, the vertex is a maximum.

The vertex is the point between the roots.

To find the roots, just make the revenue 0 and solve for p.

$-2p^2 + 1000p = 0$

p(-2p + 1000) = 0

p = 0

-2p + 1000 = 0

-2p = -1000

Since -2 times 500 = -1000

p = 500

The vertex is between 0 and 500. It is 250

A price of 250 dollars will maximize the revenue.

Revenue = p (-2p + 1000)

= 250 (-2 times 250 + 1000)

= 250 (-500 + 1000)

= 250 (500)

= 125000

61.

Your friends say that he has $2.40 in equal numbers of quarters, dimes, and nickels. How many of each coin does he have?

Trick: Let x be the number of quarters. Since the number of dimes is the same as the number of quarters, let x be the number of dimes. By the same token, let x be the number of nickels.

1 quarter = 0.25 dollar

1 dime = 0.10 dollar

1 nickel = 0.05 dollar.

x times 0.25 + x times 0.10 + x times 0.05 = 2.40

$0.25x + 0.10x + 0.05x = 2.40$

$0.40x = 2.40$

$$x = \frac{2.40}{0.40}$$

$x = 6$

Indeed, 6 quarters = $1.50

6 dimes = 0.60

6 nickels = 0.30

1.50 + 0.60 + 0.30 = 1.50 + 0.90 = 2.40

62.

I am a two-digit number whose digit in the tenth place is 1 less than twice the digit in the ones place. When the digit in the tenth place is divided by the digit in the ones place, The quotient is 1 and the remainder is 4. What number am I?

Trick: Let xy be the two-digit number with y representing the digit in the ones place and x representing the digit in the tenths place.
Then xy = 10x + y

Digit in the tenth place is 1 less than twice the digit in the ones place.

x = 2y – 1

digit in the tenth place is divided by the digit in the ones place, The quotient is 1 and the remainder is 4.

$$\begin{array}{r|l} & \ \ 1 \\ \hline y & \ \ x \\ & \ \ 4 \end{array}$$

x = 1 times y + 4

x = y + 4

You just have to solve these equations

x = y + 4

x = 2y – 1

When two quantities are equal to the same thing, they are equal to each other.

Since y + 4 and 2y – 1 are both equal to x, they are equal to each other.

$2y - 1 = y + 4$

$2y - y - 1 = y - y + 4$

$y - 1 = 4$

$y - 1 + 1 = 4 + 1$

$y = 5$

$x = y + 4 = 5 + 4 = 9$

The number is xy or 95

63.

A two-digit number is formed by randomly selecting from the digits 2, 4, 5, and 7 without replacement. What is the probability that a two-digit number contains a 2 or a 7?

Trick: When you pick the first number, the choices are 2, 4, 5,and 7. If the first number you pick is 2, then the second time you pick, it could be 4, 5, or 7 since you cannot put the 2 back. Do the same reasoning if the first number you pick is 4, 5, or 7.

Choices for first number picked	Choices for Second number picked
2	4, 5, or 7
4	2, 5, or 7
5	2, 4, or 7
7	2, 4, or 5

After you consider all cases, here is your list

24	25	27
42	45	47
52	54	57
72	74	75

To find the probability that a number contains a 2 or a 7, just count all number that has either a 2 or a 7. These are shown in blue below.

24	25	27
42	45	47
52	54	57
72	74	75

Probability that a number contains a 2 or a 7 = $\frac{10}{12}$ = 0.8333 = 83.33%

64.

Suppose you interview 30 females and 20 males at your school to find out who among them are using an electric toothbrush. Your survey revealed that only 2 males use an electric toothbrush while 6 females use it. What is the probability that a respondent did not use an electric toothbrush given that the respondent is a female?

Trick: The word given that tells you that you are dealing with a conditional probability.

So use this formula. Given two events A and B from a sample space with P(A) not equal to zero.

P(B/A) = $\dfrac{P(A \wedge B)}{P(A)}$

We can organize the information to make it easier to understand

	Use electric toothbrush	Does not use electric toothbrush
Male	2	18
Female	6	24

Let A = female

Let B = does not use electric toothbrush

P(B/A) = $\dfrac{P(A \wedge B)}{P(A)}$

P(A) = $\dfrac{30}{50}$

$A \wedge B$ is the number of females who do not use electric toothbrush.

This is equal to 24

$P(A \wedge B) = \frac{24}{50}$

P(B/A) = $\dfrac{\frac{24}{50}}{\frac{30}{50}} = \frac{24}{50} \times \frac{50}{30} = \frac{24}{30}$ = 0.80 = 80%

65.

An employer pays 15 dollars per hour plus an extra 5 dollars per hour for every hour worked beyond 8 hours up to a maximum daily wage of 220 dollars. Find a piecewise function that models this situation.

Trick:Understand what a piecewise function is. A piecewise function is a function that behaves different based on the value of x. In other words, the function will have different expressions based of the value of x.
For example, in this problem, there will be 3 different expressions for the function.

Let x be the number of hours worked.

1) Hours up to 8

2) Hours between 8 and x (wage is still less than 220)

3) Hours is bigger than x or equal to x (maximum wage is reached)

Let x be the number of hours worked.

Let us solve 1) to see what the function will look like for up to 8 hours.

If you work x hours for 15 dollars per hour, then your wage is 15x for up

to 8 hours or $0 \le x \le 8$

Now, let us solve 3)

How many extra extra hours will give a maximum wage of 220

Regular wage plus extra wage = 220

$8 \times 15 + 20x = 220$

$120 + 20x = 220$

$120 - 120 + 20x = 220 - 120$

$20x = 100$

Since 20 times 5 = 100, $x = 5$

This means that once you have worked 8 + 5 hours or 13 hours or more than 13 hours, you will be paid 220 dollars.

We can now do 2) by looking for a function that models your wage when your hours are 9, 10, 11, or 12.

Notice that once you have worked 8 hours, your wages could be

$120 + 20(1) = 120 + 20(9 - 8)$

$120 + 20(2) = 120 + 20(10 - 8)$

$120 + 20(3) = 120 + 20(11 - 8)$

$120 + 20(4) = 120 + 20(12 - 8)$

Just let x = 9, 10, 11, or 12 and you have a general format for this case.

$120 + 20(x - 8)$

Next we summarize our result

Let w(x) be the wage

$$w(x) = \begin{cases} 15x \text{ for } 0 \le x \le 8 \\ 120 + 20(x - 8) \text{ for } 8 < x < 13 \\ 220 \text{ for } x \ge 13 \end{cases}$$

66.

Divide me by 7, the remainder is 5. Divide me by 3, the remainder is 1 and my quotient is 2 less than 3 times my previous quotient. What number am I?

Trick: Let q1 be the quotient when the number is divided by 7. Let q2 be the quotient when the number is divided by 3. q2 is the bigger quotient since it is divided by a smaller number.

q2 = 3q1 – 2

Let x be the number

```
   q2  r 1                           q1   r 5
  ______                            ______
3 |  x                           7 |  x
```

x = 3q2 + 1

x = 7q1 + 5

Since the number is the same, 3q2 + 1 = 7q1 + 5

Replace $q2$ with $3q1 - 2$

$3(3q1 - 2) + 1 = 7q1 + 5$

$9q1 - 6 + 1 = 7q1 + 5$

$9q1 - 5 = 7q1 + 5$

$9q1 - 5 + 5 = 7q1 + 5 + 5$

$9q1 = 7q1 + 10$

$9q1 - 7q1 = 7q1 - 7q1 + 10$

$2q1 = 10$

Since 2 times 5 = 10, $q1 = 5$

$x = 7q1 + 5 = 7$ times $5 + 5 = 40$

67.

A company making luggages have these requirements to follow. The length is 10 inches greater than the depth and the sum of length, width, and depth may not exceed 50 inches. What is the maximum value for the depth if the manufacturer will only use whole numbers?

Trick: The word length, depth, and width indicate that we are dealing with a 3-dimensional figure. When dealing with a 3-dimensional figure, we are usually interested in finding its volume. A good idea is to try to find an equation for the volume. Also notice that the problem is asking for maximum value for the depth. It does not ask to find the greatest volume.

In fact the maximum value of x may not give the maximum volume.

Let x be the depth. Then, length = 10 + x

length + depth + width may not exceed 50.

In other words, length + depth + width is at the most 50

We can then write length + depth + width = 50

$x + 10 + x + \text{width} = 50$

$\text{width} = 50 - x - x - 10$

$\text{width} = 40 - 2x$

Volume = depth x length x width

$= x(10 + x)(40 - 2x)$

since we are dealing with a volume, all dimensions must be bigger than 0

$x > 0$

$10 + x > 0$

$40 - 2x > 0$

$10 + x > 0$

$10 - 10 + x > 0 - 10$

$x > -10$

$40 - 2x > 0$

$40 - 2x + 2x > 0 + 2x$

$40 > 2x$

$2x < 40$

Divide both sides by 2

$2x / 2 < 40 / 2$

$x < 20$

We have $x > 0$, $x > -10$, and $x < 20$

Since the dimensions must be greater than 0, $x > -10$ will be excluded.

We get $0 < x < 20$

Therefore, the maximum value for x or the depth is $x = 19$

68.

To make an open box, a man cuts equal squares from each corner of a sheet of metal that is 12 inches wide and 16 inches long. Find an expression for the volume in terms of x.

Trick: Make a drawing to see what is going on.

16

12

Let x be the length of the side of the square.

Then, the height of the box is x

The length of the box = 16 – x – x = 16 – 2x

The width of the box = 12 – x – x = 12 – 2x

V = x(16 – 2x)(12 - 2x)

69.

Ten candidates are running for president, vice-president, and secretary in the students government. You may vote for as many as 3 candidates. In how many ways can you vote for 3 or fewer candidates?

Trick: You may vote for 3 people, 2 people, 1 person, or none.

Furthermore, this is a permutation since the order does matter. Once you have a president, you do not need another president.

The same goes for vice-president, and the secretary.

Let n be the number of permutations of n items of a set arranged r times at a time.

Then the formula for permutation is $\frac{n!}{(n-r)!}$

If you vote for 3 people, the number of ways to vote = $\frac{10!}{(10-3)!}$

If you vote for 2 people, the number of ways to vote = $\frac{10!}{(10-2)!}$

If you vote for 1 person, the number of ways to vote = $\frac{10!}{(10-1)!}$

If you vote for no one, the number of ways to vote = $\frac{10!}{(10-0)!}$

$\frac{10!}{(10-3)!}$ = $\frac{10\times9\times8\times7!}{7!}$ = 10 x 9 x 8 = 90 x 8 = 720

$\frac{10!}{(10-2)!}$ = $\frac{10\times9\times8!}{8!}$ = 10 x 9 = 90

$\frac{10!}{(10-1)!}$ = $\frac{10\times9!}{9!}$ = 10

$\frac{10!}{(10-0)!}$ = $\frac{10!}{10!}$ = 1

Notice that voting for no one still count as 1 choice. It is indeed a choice that you have to vote for no one.

Total numbers of ways to cast your ballot = 720 + 90 + 10 + 1 = 821.

70.

The half-life of a medication prescribed by a doctor is 6 hours. How many mg of this medication is left after 78 hours if the doctor prescribed 100 mg?

Trick: Find out how many half-lives there are after 78 hours.

1 half- live ---------------- 6 hours

x half-lives --------------- 78 hours

To get from 6 hours to 78 hours, we need to multiply 6 by 13

Therefore, there are 13 half-lives in 78 hours.

The amount that is left after 6, 12, and 18 is shown below.

Look at it carefully so you can see the pattern

After 6 hours, there are 100 x $\frac{1}{2}$ 1 half-life

After 12 hours, there are 100 x $\frac{1}{2}$ x $\frac{1}{2}$ = 100 x $\left(\frac{1}{2}\right)^2$ 2 half-lives

After 18 hours, there are 100 x $\frac{1}{2}$ x $\frac{1}{2}$ x $\frac{1}{2}$ = 100 x $(\frac{1}{2})^3$ 3 half-lives

------

After 78 hours, there are 100 x $(\frac{1}{2})^{13}$ 13 half-lives

Amount left = 100 x 0.00012207031 mg

Amount left = 0.012207 mg

71.

Suppose you roll a red number cube and a yellow number cube.
Find P(red 2, yellow 2) and the probability to get any matching pairs of numbers.

Trick: Do not let the color of the cube confuse you. Just make a list of all possible outcomes.

The cubes can give the following outcomes separately

red 1	yellow 1
red 2	yellow 2
red 3	yellow 3
red 4	yellow 4
red 5	yellow 5
red 6	yellow 6

There will be a total of 36 outcomes then

(red 1 yellow 1)	(red 2 yellow 1)	(red 3 yellow 1)
(red 1 yellow 2)	(red 2 yellow 2)	(red 3 yellow 2)
(red 1 yellow 3)	(red 2 yellow 3)	(red 3 yellow 3)
(red 1 yellow 4)	(red 2 yellow 4)	(red 3 yellow 4)
(red 1 yellow 5)	(red 2 yellow 5)	(red 3 yellow 5)
(red 1 yellow 6)	(red 2 yellow 6)	(red 3 yellow 6)

(red 4 yellow 1)	(red 5 yellow 1)	(red 6 yellow 1)
(red 4 yellow 2)	(red 5 yellow 2)	(red 6 yellow 2)
(red 4 yellow 3)	(red 5 yellow 3)	(red 6 yellow 3)
(red 4 yellow 4)	(red 5 yellow 4)	(red 6 yellow 4)
(red 4 yellow 5)	(red 5 yellow 5)	(red 6 yellow 5)
(red 4 yellow 6)	(red 5 yellow 6)	(red 6 yellow 6)

(red 2, yellow 2) happens only once as shown in red above.

P(red 2, yellow 4) = $\frac{1}{36}$

Matching pairs of numbers are shown in blue and red. There are 6 of them.

probability to get any matching pairs of numbers = $\frac{6}{36} = \frac{1}{6}$

72.

A movie theater in a small town usually open its doors 3 days in a row and then closes the next day for maintenance. Another movie theater 3 miles away open 4 days in a row and then closes the next day for the same reason. Suppose both movie theaters are closed today and today is Wednesday, when is the next time they will both be closed again on the same day?

Trick: One of the movie theater opens 3 days in a row and then closes on the 4^{th} day. The other movie theater opens 4 days in a row and then closes on the 5^{th} day. So one closes on the 4^{th} day while the other closes on the 5^{th} day. To solve this kind of problem, you need to look for the least common multiple.

To find the time when they will be both closed, we need to find the

LCM(4, 5)

Multiples of 4: 4, 8, 12, 16, **20**, 24, 28, 32

Multiples of 5: 5, 10, 15, **20**, 25, 30

The LCM(4, 5) = 20.

Both of them will be closed on the 20^{th} day.

They were both closed on Wednesday.

Thursday-Wednesday is 7 days

Thursday-Wednesday again is 14 days

Thursday-Tuesday is 20 days.

They will be both closed on Tuesday

73.

An investor invests 5000 dollars at 10% and the rest at 5%. How much was invested at 5% if the yield is one-fifth of the amount invested at 10%?

Trick: Amount invested at 5% is x + 5000. Once you find x, add that to 5000 to find the amount invested at 5%.

interest for 5000 investment + interest for (x + 5000) = one-fifth of 5000

5000 times 10% + (x + 5000)times 5% = 5000/5

5000 times 0.10 + 5% times x + 5000 times 5% = 1000

$500 + 0.05x + 250 = 1000$

$500 + 250 + 0.05x = 1000$

$750 + 0.05x = 1000$

$750 – 750 + 0.05x = 1000 - 750$

$0.05x = 250$

$$x = \frac{250}{0.05}$$

$x = 5000$

x + 5000 = 5000 + 5000 = 10000. Thus, 10000 dollars was invested at 5%

74.

20000 students took a standardized math test. The scores on the test are normally distributed, with a mean score of 85 and a standard deviation of 5. About how many students scored between 90 and 95?

Trick: We are dealing with a normal distribution and this is the key concept.When a data set is normally distributed, about 68% of the data fall within one standard deviation of the mean. About 95% of the data fall within two standard deviations of the mean.

What do we do with the standard deviation? Just add the value to the mean to find scores above the mean. And subtract to find scores below the mean

Let 1SD = 1 standard deviation, 2SD = 2 standard deviations

Mean

75 80 85 90 95

1SD

34% 34%

34% + 34% = 68%

2SD

13.5% 13.5%

13.5 + 68% + 13.5% = 95%

About 13.5% of students have a score between 90 and 95.

13.5% of 20000 = 0.135 x 20000 = 2700

Therefore, about 2700 students scored between 90 and 95

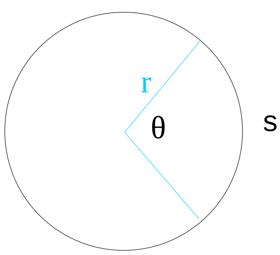
75.

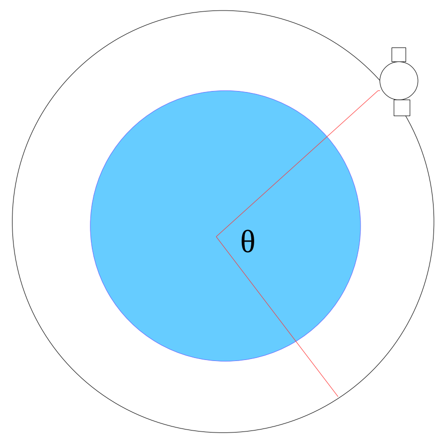
A satellite, located 2400 km above Earth's surface, is in circular orbit around the earth. If it takes the satellite 3 hours to complete 1 orbit, how far is the satellite after 1 hour?

Trick: Make a drawing to describe the situation and use the formula to find the length of an intercepted arc called s. Keep in mind too that θ is in radians.

Know that the radius of the Earth is 6400 km

Let s be the minor arc formed by the raddi then, s = rθ.





Let us use s = rθ for our problem.

r is the distance from the center of the Earth to the satellite.

r = radius of the earth + distance from Earth's surface to satellite

r = 6400 + 2400 = 8800 km

Now find θ. 1 hour is only one-third of the orbit.

1 orbit is a full circle or 2π in radians

one-third of the orbit is $2\pi/3 = (2 \text{ times } 3.14)/3 = 6.28/3 = 2.0933$

$s = 8800 \times (2\pi/3)$

$s = 8800 \times 2.0933$

$s = 18421.04$ km

76.

In a group of 10 people, what is the probability that at least two people in the group have the same birthday?

Trick:

Let A be the probability that at least two people have the same birthday.

Let B be the probability that no two people have the same birthday.

Notice that $P(A) + P(B) = 1$

It is easier to find first the probability that no two people in the group have the same birthday or $P(B)$.

Once we find $P(B)$, We can say $P(A) = 1 - P(B)$

Let us then find $P(B)$. To begin we need to reason as shown in the following to ensure that no two people have the same birthday.

The first person can choose from 365 days. The second person can choose from 364 days, the third person can choose from 363 days, and so forth all the way to tenth person.

We can use the multiplication counting principle to find the number of ways this can be done.

It is 365 x 364 x 363 x 362 x 361 x 360 x 359 x 358 x 357 x 356.

Now, we need to use the multiplication counting principle again to find the number of ways birthdays can be selected.

For each person there are 365 ways to choose a birthday. For all ten, there are 365 x 365 x 365 x 365 x 365 x 365 x 365 x 365 x 365 x 365

P(B) = $\frac{365 \times 364 \times 363 \times 362 \times 361 \times 360 \times 359 \times 358 \times 357 \times 356}{365 \times 365 \times 365 \times 365 \times 365 \times 365 \times 365 \times 365 \times 365 \times 365}$

P(B) = $\frac{365}{365}$ X $\frac{364}{365}$ X $\frac{363}{365}$ X $\frac{362}{365}$ X $\frac{361}{365}$ X $\frac{360}{365}$ X $\frac{359}{365}$ X $\frac{358}{365}$ X $\frac{357}{365}$ X $\frac{356}{365}$

P(B) = 1 X 0.997 X 0.994 X 0.992 X 0.989 X 0.986 X 0.983 X 0.98 X 0.978 X 0.975

P(B) = 0.881

P(A) = 1 – 0.881

P(A) = 0.119

77.

During a fundraising for cancer at a gala, everybody shakes hands with everyone else in the room before the event and after the event is finished. If n people attended the gala, how many different handshakes occur?

Trick: Solve a simpler problem and look for a pattern. For instance, find out how many handshakes for 3, 4, or 5 people and try to find a pattern.

Let us use 3 smiley faces to represent 3 people shaking hands.

Blue guy shakes hand with orange guy   1 handshake

Blue guy shakes hands with green guy   1 handshake

Orange guy shakes hands with green guy   1 handshake

So the blue guy will shake hand with everybody on the right or 2 people.

Then, the orange guy will shake hands with everybody on the right or 1 person.

Thus, for 3 people, there are 2 + 1 handshakes.

Let us add one more person.

Blue guy shakes hands with orange guy, green guy, and yellow guy

3 handshakes

Orange guy shakes hands with green guy and yellow guy

2 handshakes

Green guy shakes hands with yellow guy

1 handshake

Thus, for 4 people, there are 3 + 2 + 1 handshakes.

Do you see a pattern?

For 3 people, there are 2 + 1 or 3 handshakes

For 4 people, there are 3 + 2 + 1 or 6 handshakes

For 5 people, there are 4 + 3 + 2 + 1 or 10 handshakes

Now, we need to make an important observation!

For 3 people, you can get 3 handshakes by doing $\frac{3\times 2}{2}$

For 4 people, you can get 6 handshakes by doing $\frac{4\times 3}{2}$

For 5 people, you can get 10 handshakes by doing $\frac{5\times 4}{2}$

So here is what I am seeing $\frac{number\,of\ people\ \times\ (number\,of\ people\ -1)}{2}$

Let n be the number of people,then the number of handshakes before the gala is

$$\frac{n\ \times\ (n\ -1)}{2}$$

However, recall that they shake hands before and after the gala. Therefore, we need to multiply our expression by 2

$\frac{n \times (n-1)}{2}$ x 2 = n(n – 1)

Therefore, the total number of handshakes is n(n – 1) where n is the number of people.

78.

Two cubes have side lengths that are equal to 2x and 4x. How many times greater than the surface of the small cube is the surface area of the large cube?

Trick: Find the surface area of both cubes and do a ratio of the surface area of the big cube to the surface area of the small cube.

Formula to find the surface area of a cube

Surface area = $6s^2$ where s is the length of one side.

Surface area of small cube = $6s^2$ = $6(2x)^2$ = $6(4x^2)$ = $24x^2$

Surface area of big cube = $6s^2$ = $6(4x)^2$ = $6(16x^2)$ = $96x^2$

Ratio of surface area of big cube to surface area of small cube is

$$\frac{96x^2}{24x^2} = 4$$

The surface area of the bigger cube is 4 times bigger than the surface area of the smaller cube when the length of the side is doubled or goes from 2x to 4x.

79.

Suppose you have a job in a restaurant that pays $8 per hour. You also have a job at Walmart that pays $10 per hour. You want to earn at least 200 per week. However, you want to work no more than 25 hours per week . Show 3 different ways you could work at each job.

Trick:

Let x be the time you spend working at the restaurant

Let y be the number of hours you work at Walmart.

$$Total\ hours\ work\ per\ week \leq 25$$

$$Total\ earnings\ per\ week \geq 200$$

We then have the following two equations to graph

$$x + y \leq 25$$

$$8x + 10y \geq 200$$

First, we need to graph x + y = 25 and 8x + 10y = 200

We can easily graph x + y = 25 and 8x + 10y = 200 by looking for the x-intercepts and the y-intercepts.

$x + y = 25$

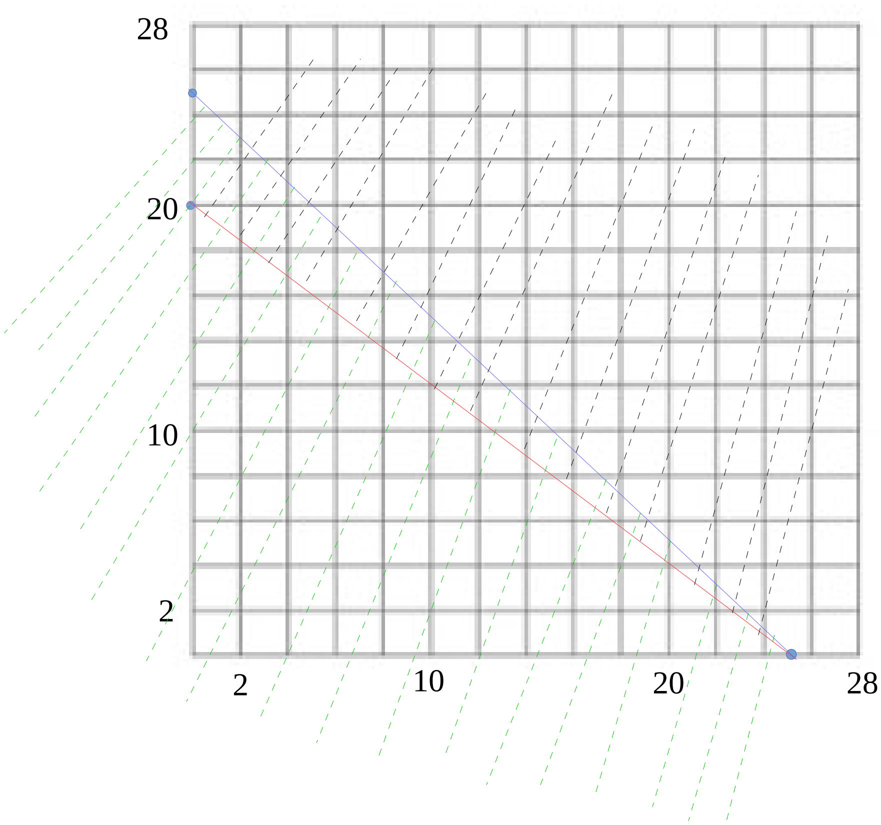
Let $x = 0$, then $y = 25$. y-intercept is (0, 25)

Let $y = 0$, then $x = 25$. x-intercepts is (25, 0)

$8x + 10y = 200$

Let $x = 0$, then $10y = 200$ and $y = 20$. y-intercept is (0, 20)

Let $y = 0$, then $8x = 200$ and $x = 25$. x-intercepts is (25, 0)



For the blue graph, we shade below the blue graph because if we choose (10, 14) as a test point, $10 + 14 \leq 25$

We shade everything below the blue graph with green dashed lines.

For the red graph, we shade above the red graph it because if we choose (12, 16) as a test point, $8 \times 12 + 10 \times 16 \geq 200$

We shade everything above the red graph with blacked dashed lines.

The solution is any point where you see black dashed lines and green dashed lines at the same time. Also you can pick any point on the blue line or the red line.

Let us see if we can find 3 different points in that area.

Pick (2, 20), (8, 16), and (18, 6). They should solve both inequalities below at the same time.

For instance, (8, 16) means that you can work 8 hours at the restaurant and 16 hours at Walmart.

$x + y \leq 25$

$8x + 10y \geq 200$

Let us check one of them

(2, 20) $2 + 20 = 22$ and $22 \leq 25$

$8 \times 2 + 10 \times 20 = 16 + 200 = 216$ and $210 \geq 200$

80.

Two company offer tutoring services. Company A realizes that when they tutor for 3 hours, they make 45 dollars. When they tutor for 7 hours they make 105 hours. Company B realizes that when they tutor for 2 hours, they make 34 dollars. When they tutor for 6 hours they make 102 hours. Assuming that the number of hours students sign for tutoring is the same for both company, which company will generate more revenue?

Trick: Find the slope. This will give you the amount of money each company makes per hour.

Company A

(3, 45) (7, 105)

m = $\frac{105-45}{7-3}$ = $\frac{60}{4}$ = $\frac{15}{1}$

Company B

(2, 34) (6, 102)

m = $\frac{102-34}{6-2}$ = $\frac{68}{4}$ = $\frac{17}{1}$

Company A makes 15 dollars per hour while company B makes 17 dollars per hour. Company B is likely to generate more revenue.

81.

You want to fence a rectangular area for kids in the backyard. To save on fences, you will use the back of your house as one of the four sides. Find the possible dimensions if the house is 60 feet wide and you want to use at least 160 feet of fencing.

Trick: Perimeter is at least 160 feet. Since the house is 60 feet wide, the width or w should be no more than 60 feet.

Let L be the length, we then have the following two inequalities.

$w \leq 60$

$w + 2L \geq 160$

We will put w on the x-axis and the L on the y-axis

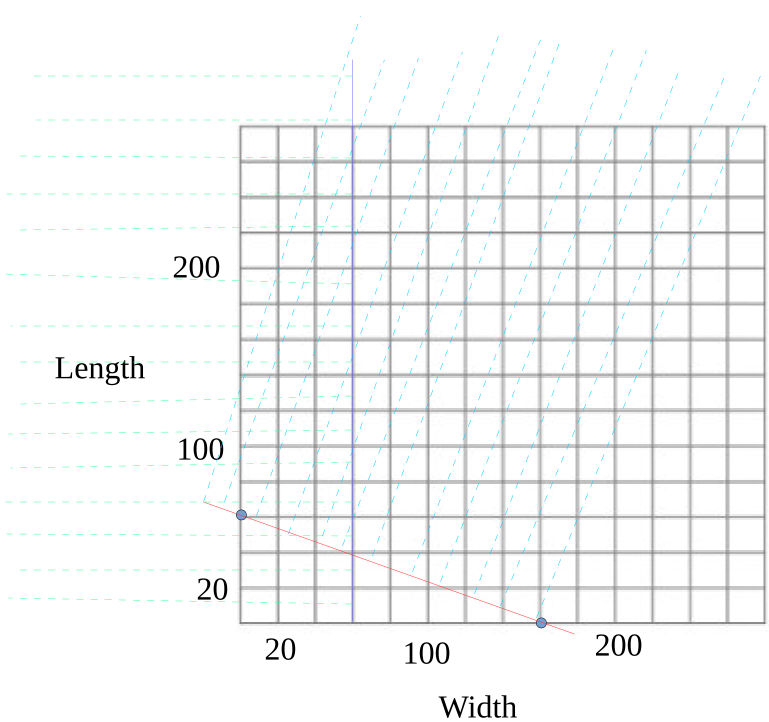
We just need to graph w = 60 and w + 2L = 160

W = 60 is the blue line.

Let us graph w + 2L = 160 by looking for x-intercept and y-intercept.

If L = 0, w = 160. x-intercept is (160, 0)

If w = 0,   2L = 160 and L = 80. y-intercept is (0, 80)



Since $w \le 60$, we shade everything on the left of the blue line.
For the red line, we shade everything above the red line since any point above the red line will make the inequality $w + 2L \ge 160$ true.

Just choose any point where you see sky blue dashed lines and green dashed lines at the same time. You can also choose a point on the blue line as long as it is above the red line. You can also choose a point on the red line as long as it is on the left of the blue line. You could also choose the point at the intersection of the blue line and the red line as long as you look for it. For example, you could choose (20, 80) or (60, 220)

82.

When a number in increased by 20%, the result is the same when it is decreased by 10% plus 12. What is the number?

Trick: Let x be the number. x times 20% = x times 0.20 = 0.20x

Increased by 20% means x + 0.20x

Decreased by 10% means x – 0.10x

Decreased by 10% + 12 means x – 0.10x + 12

x + 0.20x is the same as x – 0.10x + 12

x + 0.20x = x – 0.10x + 12

1.20x = 0.90x + 12

1.20x – 0.90x = 12

0.30x = 12

$x = \frac{12}{0.30}$

x = 40

83.

The average of three numbers is 47. The biggest number is five more than twice the smallest. The range is 35. What are the three numbers?

Trick: Make sure you clearly identify the numbers. So, let x be the smallest number, let z be the biggest number, and Let y be the last number. Also remember that the range is biggest number minus smallest number.

We then have the following equations

$\frac{x+y+z}{3} = 47$ (1) $\qquad$ z – x = 35 (2) $\qquad$ z = 2x + 5 (3)

We can solve for x by replacing the value of z or 2x + 5 in (2)

2x + 5 – x = 35

2x – x + 5 = 35

x + 5 = 35

x + 5 – 5 = 35 – 5

x = 30

z = 2x + 5 = 2 x 30 + 5 = 60 + 5 = 65

To get y, use (1) or $\frac{x+y+z}{3}$ = 47

$\frac{30+y+65}{3}$ = 47

$\frac{y+95}{3}$ = 47

Multiply both sides by 3

3 x $\frac{y+95}{3}$ = 47 x 3

y + 95 = 141

y + 95 – 95 = 141 – 95

y = 46

The three numbers are 30, 46, and 65

84.

The percent of increase of a number from its original amount to 36 is 80%. What is the original amount of the number?

Trick: Remember how percent of increase is calculated.

Percent of increase = $\frac{Final\ \ amount\ \ -\ \ original\ \ amount}{original\ \ amount}$

Let x be the original amount and let 36 be the final amount.

$$80\% = \frac{36 - x}{x}$$

$$\frac{80}{100} = \frac{36 - x}{x}$$

Cross multiplying

80 times x = 100 times (36 – x)

80x = 100 times 36 – 100 times x

80x = 3600 – 100x

80x + 100x = 3600 – 100x + 100x

180x = 3600

$$x = \frac{3600}{180} = \frac{360}{18} = 20$$

85.

When Peter drives to work, he averages 45 miles per hour because of traffic. On the way back home, he averages 60 miles per hour because traffic is not as bad. The total travel time is 2 hours. How far is Peter’s house from work?

Trick: distance = speed x time

We have the speed, we just need the find either the time from home to work or from work to home. Assume that the distance from home to work is the same as the distance from work to home.

Let t be the time it takes John to drive from home to work

Let s be the time it takes John to drive from work to home.

On the way to work, d = 45 x t

On the way home, d = 60 x s

t + s = 2

s = 2 - t

Since the distance is the same, we get the following equation

45t = 60(2 – t)

45t = 60 times 2 – 60 times t

45t = 120 – 60t

45t + 60t = 120 – 60t + 60t

105t = 120

$t = \frac{120}{105} = 1.143$

d = 45 times 1.143 = 51.43 miles. Work is 51.43 miles away.

86.

An advertising company takes 20% from all revenue that it generates for its affiliates. If the affiliates were paid 15200 dollars this month, how much revenue did the advertising company generate this month?

Trick: Revenue generated minus revenue taken by advertising company = revenue paid to affiliates

Let x be the revenue that the advertising company generated this month.

x – 20% of x = 15200

$x - 0.20x = 15200$

$0.80x = 15200$

$$x = \frac{15200}{0.80}$$

$x = 19000$

87.

A company's revenue can be modeled with a quadratic equation. The company noticed that when they sell either 2 or 12 items, the revenue is 0. How much is the revenue when they sell 20 items?

Trick: Find the quadratic model for the company’s revenue by working backward.

Recognize that 2 and 12 are the zeros of the quadratic equation.

The zeros are found when we solve (x – 2)(x – 12)

Just do the multiplication and it will lead to the quadratic equation.

(x – 2)(x – 12) = x times x + x times -12 + -2 times x + -2 times – 12

= x^2 + -12x + -2x + 24

= x^2 + -14x + 24

The revenue function is f(x) = x^2 + -14x + 24

f(20) = 20^2 + -14 times 20 + 24

= 400 – 280 + 24

= 120 + 24

= 144

The company will make 144 dollars when it sells 20 items.

88.

A ball bounced 4 times, reaching three-fourths of its previous height with each bounce. After the fourth bounce, the ball reached a height of 25 cm. How high was the ball when it was dropped?

Trick: Let x be the height of the ball when it was dropped. Break the problem down to show height after first bounce, second bounce, third bounce, and fourth bounce.

Keep in mind that three-fourths of anything = three-fourths times anything.

First bounce: Height is three-fourths of its previous height or $\frac{3}{4}$ of x

or $\frac{3}{4}$ times x = $\frac{3}{4}x$

Second bounce: Height is three-fourths of its previous height or $\frac{3}{4}$ of

$\frac{3}{4}x$ or $\frac{3}{4}$ times $\frac{3}{4}x$ = $\frac{9}{16}x$

Third bounce: Height is three-fourths of its previous height or $\frac{3}{4}$ of

$\frac{9}{16}x$ or $\frac{3}{4}$ times $\frac{9}{16}x$ = $\frac{27}{64}x$

Fourth bounce: Height is three-fourths of its previous height or $\frac{3}{4}$ of

$\frac{27}{64}x$ or $\frac{3}{4}$ times $\frac{27}{64}x$ = $\frac{81}{256}x$

The height of $\frac{81}{256}x$ is the same as 25 cm

$$\frac{81}{256}x = 25$$

Multiply both sides by $\frac{256}{81}$

$$\frac{256}{81} \text{ times } \frac{81}{256}x = \frac{256}{81} \text{ times } 25$$

$$x = \frac{256 \times 25}{81} = \frac{6400}{81} = 79.0123$$

The height of the ball when it was dropped is 79.0123

89.

A rental company charges 40 dollars per day plus $0.30 per mile. You rent a car and drop it off 4 days later. How many miles did you drive the car if you paid 325.5 dollars which included a 5% sales tax?

Trick: Let x be the number of miles you drove the car.

Cost to rent the car plus sales tax = total cost

Cost to rent the car = cost for 4 days plus cost of mileage

= 40 times 4 + 0.30 times x

= 160 + 0.30x

Cost of sales tax is calculated from the cost to rent the car or

$160 + 0.30x$

Cost of sales tax = $(160 + 0.30x)$ times 5%

Total cost = $160 + 0.30x + (160 + 0.30x)$ times 5%

The total cost is also equal to 325.5

$160 + 0.30x + (160 + 0.30x)$ times $5\% = 325.5$

$160 + 0.30x + (160 + 0.30x)$ times $0.05 = 325.5$

$160 + 0.30x + 160$ times $0.05 + 0.30x$ times $0.05 = 325.5$

$160 + 0.30x + 8 + 0.015x = 325.5$

$160 + 8 + 0.30x + 0.015x = 325.5$

$168 + 0.315x = 325.5$

$168 - 168 + 0.315x = 325.5 - 168$

$0.315x = 157.5$

$$x = \frac{157.5}{0.315}$$

$$x = 500$$

You drove 500 miles with the car.

90.

Two students leave school at the same time and travel in opposite directions along the same road. One walk at a rate of 3 mi/h. The other bikes at a rate of 8.5 mi/h. After how long will they be 23 miles part?

Trick: Let d1 be the distance the first student travels. Let d2 be the distance the other student travels. Let t be the time it takes them to be 23 miles apart.

d1 + d2 = 23

d1 = vt = 3t   d2 = vt = 8.5t

3t + 8.5t = 23

11.5t = 23

t = 23 / 11.5 = 2

It will them 2 hours to be 23 miles apart.

91.

Brown has the same number of brothers as sisters. His sister Sylvia as twice as many brothers as sisters. How many children are in the family?

Trick: Use of logic

Suppose Brown has   1 sister   1 brother

This scenario will not work because Sylvia has no sister.

Suppose Brown has 2 sisters 2 brothers

Sylvia then has 1 sister and 3 brothers. This will not work either since Sylvia has 3 times as many brothers as sisters.

Suppose Brown has 3 sisters 3 brothers

Sylvia then has 2 sisters and 4 brothers. This will work this time since Sylvia has 2 times as many brothers as sisters.

Therefore, there are 7 children in the family.

92.

Ethan has the same number of male classmates as female classmates. His classmate Olivia has three-fourths as many female classmates as male classmates. How many students are in the class?

Trick: Use of logic. Let FC = female classmate and MC = male classmate.

Suppose Ethan has 1 FC 1 MC

This scenario will not work because Olivia has no classmates

Suppose Ethan has 2 FC 2 MC

Olivia has 1 FC and 3 MC

$\frac{3}{4}$ of 3 = 2.25

This scenario does not work since 1 female classmate is not equal to three-fourths of 3

Suppose Ethan has 3 FC 3 MC

Olivia has 2 FC and 4 MC

$\frac{3}{4}$ of 4 = 3

This scenario does not work since 2 female classmates is not equal to three-fourths of 4 or 3

Continuing in this pattern,

When Ethan has	Olivia will have
4 FC and 4 MC	3 FC and 5 MC
5 FC and 5 MC	4 FC and 6 MC
6 FC and 6 MC	5 FC and 7 MC
7 FC and 7 MC	6 FC and 8 MC

Since 6 female classmates is $\frac{3}{4}$ of 8, the last row is the right combination.

Therefore, the number of students in the class in 15.

93.

Noah wants to share a certain amount of money with 10 people. However, at the last minute, he is thinking about decreasing the amount by 20 so he can keep 20 for himself and share the money with only 5 people. How much money is Noah trying to share if each person still gets the same amount?

Trick: Let x be the amount he wants to share. There are 2 cases.

Noah can share the money with 10 people. So each person will get $\frac{x}{10}$

Noah can decrease the amount by 20 and share the money with 5 people. So each person will get $\frac{x-20}{5}$

If each person still gets the same amount, then $\frac{x}{10} = \frac{x-20}{5}$

Just solve $\frac{x}{10} = \frac{x-20}{5}$

Cross multiply.

5 times x = 10(x – 20)

5x = 10x – 200

5x – 5x = 10x – 5x – 200

0 = 5x – 200

0 + 200 = 5x – 200 + 200

200 = 5x

Since 5 times 40 = 200, x = 40

The amount of money Noah wants to share is 40 dollars.

Indeed, $\frac{40}{10}$ = $\frac{40-20}{5}$ = 4

94.

The square root of me plus the square root of me is me. Who am I?

Trick : Let me be x

Then, $\sqrt{x}+\sqrt{x}=x$

$2\sqrt{x}=x$

Raise both sides to the second power

$(2\sqrt{x})^2=x^2$

$2^2(\sqrt{x})^2=x^2$

$4x=x^2$

$4x-4x=x^2-4x$

$0=x^2-4x$

x(x – 4) = 0

x = 0 and x – 4 = 0 or x = 4 Indeed $\sqrt{4}+\sqrt{4}=4$ or 2 + 2 = 4

95.

A cash drawer contains 160 bills, all 10s and 50s. If the total value of the 10s and 50s is $1,760. How many of each type of bill are in the drawer?

Trick:

Number of 10s + number of 50s = 160

amount of 10s + amount of 50s = 1760

Let x be the number of 10s and let y be the number of 50s

$$x + y = 160 \qquad (1)$$

$$10x + 50y = 1760 \qquad (2)$$

Solve for y in (1)

$$x + y = 160$$

$$x - x + y = 160 - x$$

$$y = 160 - x$$

Replace $160 - x$ in (2)

$$10x + 50(160 - x) = 1760$$

$$10x + 8000 - 50x = 1760$$

$$10x - 50x = 1760 - 8000$$

$$-40x = -6240$$

x = -6240 / -40

x = 156

y = 160 – 156 = 4

The cash drawer has 4 50s and 156 10s

96.

You want to make 28 grams of protein snack mix made with peanuts and granola. Peanuts contain 7 grams of protein per ounce and granola contain 3 grams of protein per ounce. How many ounces of granola should you use for 1 ounce of peanuts?

Trick:

Number of protein in 1 ounce of peanuts times number of ounces of peanuts plus number of protein in 1 ounce of granola times number of ounces of granola equal 28 grams.

Let x be the number of ounces used for peanuts.

Let y be the number of ounces used for granola.

7 times x + 3 times y = 28

7x + 3y = 28

Fir 1 ounce of peanuts, x = 1

7 times 1 + 3y = 28

7 + 3y = 28

$7 - 7 + 3y = 28 - 7$

$0 + 3y = 21$

$3y = 21$

Since 3 times 7 = 21, y = 7.

For 1 ounce of peanuts, use 7 ounces of granola.

Indeed = 7 times 1 + 3 times 7 = 7 + 21 = 28.

97.

The length of a rectangular prism is quadrupled, the width is doubled, and the height is cut in half. If V is the volume of the rectangular prism before the modification, express the volume after the modification in terms of V.

Trick: In terms of V means that V has to be part of the answer. Before the modification, the dimensions are l, w, and h

Volume before the modification = L x w x h

After the modification, the dimensions are 4L, 2w, and $\frac{h}{2}$

Volume after the modification = 4L x 2w x $\frac{h}{2}$

= 8Lw x $\frac{h}{2}$

= 4 L x w x h

Since Substitute L x w x h for V

Volume after the modification = 4V

98.

A car rental has CD players in 85% of its cars. The CD players are randomly distributed throughout the fleet of cars. If a person rents 4 cars, what is the probability that at least 3 of them will have CD players?

Trick: At least 3 of them will have CD players means 3 or 4 of them will have CD players.

Let A = car will have CD player

Let B = car will have no CD player

P(A) = 85% = 0.85 and P(B) = 1 – 0.85 = 0.15

P(A and A and A and A) = P(A) x P(A) x P(A) x P(A)

= 0.85 x 0.85 x 0.85 x 0.85

= 0.52

Now, if 3 of them have CD players, then 1 of them will not.

You will have the following possibilities

BAAA, ABAA, AABA, or AAAB

BAAA means that the first car has no CD player and the next 3 cars have CD players.

We then have the following probabilities

P(BAAA) = 0.15 X 0.85 X 0.85 X 0.85 = 0.092

P(ABAA) = 0.85 X 0.15 X 0.85 X 0.85 = 0.092

P(AABA) = 0.85 X 0.85 X 0.15 X 0.85 = 0.092

P(AAAB) = 0.85 X 0.85 X 0.85 X 0.15 = 0.092

P(BAAA or ABAA or AABA or AAAB) = P(BAAA) + P(ABAA) + P(AABA) + P(AAAB)

= 0.092 + 0.092 + 0.092 + 0.092

= 0.368

Probability that 3 or 4 cars will have CD players is 0.52 + 0.368 = 0.888

99.

Jacob’s hourly wage is 4 times as much as Noah. When Jacob got a raise of 2 dollars, Noah accepted a new position that pays him 2 dollars less per hour. Jacob now earns 5 times as much money as Noah. How much money do they make per hour after Jacob got the raise?

Trick: Let x be Noah’s hourly wage before he changed position. Then, 4x is Jacob’s hourly wage before he got the raise.

After Noah accepted the new position, his hourly wage is $x - 2$

After Jacob got the raise, his hourly wage is $4x + 2$

Now Jacob's hourly wage is 5 times as much as Noah's hourly wage.

In other words, $4x + 2 = 5(x - 2)$

$4x + 2 = 5(x - 2)$

$4x + 2 = 5x - 10$

$4x - 5x + 2 = 5x - 5x - 10$

$-x + 2 = -10$

$-x + 2 - 2 = -10 - 2$

$-x = -12$

$x = 12$

Before the raise, Noah's income is $12 per hour and Jacob's is $48 per hour. And 4 x 12 = 48

After the raise, Noah's income is $10 per hour and Jacob's is $50 per hour And 5 x 10 = 50

100.

You own a catering business that makes specialty cakes. Your company has decided to create three types of cakes. To create these cakes, it takes a team that consists of a decorator, a baker, and a design consultant. Cake A takes the decorator 9 hours, the baker 6 hours, and the design consultant 1 hour to complete. Cake B takes the decorator 10 hours, the baker 4 hours, and the design consultant 2 hours. Cake C takes the decorator 12 hours, the baker 4 hour, and the design consultant 1 hour. Without hiring additional employees, there are 398 decorator hours available, 164 baker hours available, and 58 design consultant hours available. How many of each type of cake can be created?

Trick:

A = The number of cake A that can be produced.
B = The number of cake B that can be produced.
C = The number of cake C that can be produced.

To solve this problem, you will need to solve a system with 3 variables and 3 equations.

Decorator

Time it will take the decorator to make cake A, B, and C = 398 hours

If cake A can be made in 9 hours, the time it will take to make A cakes is $9 \times A$

If cake B can be made in 10 hours, the time it will take to make B cakes is $10 \times B$

If cake C can be made in 12 hours, the time it will take to make C cakes is $12 \times C$

The total time it will take the decorator to make all 3 types of cake is equal to 398

$9 \times A + 10 \times B + 12 \times C = 398$

$9A + 10B + 12C = 398$ Equation 1

Baker

Time it will take the baker to make cake A, B, and C = 164

If cake A can be made in 6 hours, the time it will take to make A cakes is $6 \times A$

If cake B can be made in 4 hours, the time it will take to make B cakes is $4 \times B$

If cake C can be made in 4 hours, the time it will take to make C cakes is $4 \times C$

The total time it will take the baker to make all 3 types of cake is equal to 164

$6 \times A + 4 \times B + 4 \times C = 164$

$6A + 4B + 4C = 164$ Equation 2

Design consultant

Time it will take the design consultant to make cake A, B, and C = 58

If cake A can be made in 1 hours, the time it will take to make A cakes is $1 \times A$

If cake B can be made in 2 hours, the time it will take to make B cakes is $2 \times B$

If cake C can be made in 1 hours, the time it will take to make C cakes is $1 \times C$

The total time it will take the baker to make all 3 types of cake is equal to 58

$1 \times A + 2 \times B + 1 \times C = 58$

$1A + 2B + 1C = 58$ Equation 3

The system of equations to solve is

$9\ A + 10B + 12C = 398$ Equation 1

$6A + 4B + 4C = 164$ Equation 2

$1A + 2B + 1C = 58$ Equation 3

Use equation 2 and equation 3 to eliminate the variable C

To achieve this, multiply equation 3 by -4

We get

$6A + 4B + 4C = 164$

$-4A + -8B + -4C = -232$

Add the two equation above

$6A + 4B + 4C = 164$

$-4A + -8B + -4C = -232$

$2A + -4B + 0 = -68$

$2A + -4B = -68$ Equation 5

Use equation 1 and equation 2 to eliminate the variable C again

To achieve this, multiply equation 2 by -3

We get

$9A + 10B + 12C = 398$

$-18A + -12B + -12C = -492$

Add the two equation above

$$\begin{array}{r} 9A + 10B + 12C = 398 \\ -18A + -12B + -12C = -492 \\ \hline -9A + -2B + 0 = -94 \end{array}$$

$-9A + -2B = -94$ Equation 6

Use equation 5 and equation 6 to eliminate the variable B

To achieve this, divide equation 5 by -2

We get

$-A + 2B = 34$

$-9A + -2B = -94$

Add the two equation above

$$\begin{array}{r} -A + 2B = 34 \\ -9A + -2B = -94 \\ \hline -10A + 0 = -60 \end{array}$$

$-10A = -60$

Since $-10 \times 6 = -60$, $A = 6$

Replace A in $-A + 2B = 34$ to get B

$-6 + 2B = 34$

$2B = 34 + 6$

$2B = 40$

Since $2 \times 20 = 40$, $B = 20$

Replace A and B in $1A + 2B + 1C = 58$ to get C

$6 + 2 \times 20 + C = 58$

$6 + 40 + C = 58$

$46 + C = 58$

Since $46 + 12 = 58$, $C = 12$

Final answer:

This situation can produce 6 cakes A, 20 cakes B, and 12 cakes C