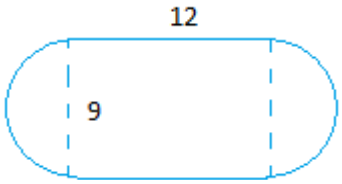
10th grade math test solution

1.

What is the perimeter and area of the following running track?



Perimeter

The perimeter of the two straight lines is 12 + 12 = 24

2 half circles is 1 circle, so just get the perimeter of 1 circle.

The formula to get the perimeter of a circle is p = 2πr

r= $\frac{9}{2}$ = 4.5

p = $2\times3.14\times4.5$ = 28.26

perimeter of the running track is 24 + 28.26 = 52.26

Area

The area of the rectangle is 9×12 = 108

The formula to get the area of a circle is A = πr^2

A = 3.14×4.5^2

= $3.14\times4.5\times4.5$

= 63.585

Total area = 108 + 63.585 = 171.585

2.

Sentence : A square is a rectangle

a. Write the sentence as a conditional and identify the hypothesis and the conclusion.

b. Write the converse of the conditional and say if the sentence is a biconditional.

c. Write the inverse and the contrapositive of the conditional and investigate the truthfulness of the

inverse and the contrapositive

A conditional is a statement that has the following format

If ..... then

a. If a figure is a square, then it is a rectangle

The hypothesis is "a figure is a square"

The conclusion is "it is a rectangle".

Notice that 'if' and 'then' were not included in the hypothesis and the conclusion.

b. The converse of the conditional is when the hypothesis becomes the conclusion and the conclusion becomes the hypothesis.

The converse of "If a figure is a square, then it is a rectangle" is "If a figure is a rectangle, then it is a square"

If both the conditional and the converse are true, then the statement is a biconditional

The conditional is true since a square is also a rectangle. However, the converse is not true.

A rectangle is not a square since the sides of a rectangle are not all equal

Therefore, the statement is not a biconditional

c. The inverse of a conditional is when you negate both the hypothesis and the conclusion

The conditional is "If a figure is a square, then it is a rectangle."

If you negate both the hypothesis and the conclusion, you get:

If a figure is not a square, then, it is not a rectangle.

Is this statement true?

Ask yourself, " If the figure is not a square, what could it be?"

Not being a square means that it does not have 4 equal sides. But it could still have all the other properties of a rectangle. Thus, this statement is false.

The contrapositive is when you switch the hypothesis and the conclusion of the inverse.

The inverse is "If a figure is not a square, then, it is not a rectangle."

The contrapositive is " If a figure is not a rectangle, then, it is not a square."

This statement is true since a figure has to be first a rectangle even before the possibility of become a square

3.

The measure of an angle is four times as large as its supplement. What is the measure of its supplement?

Let x be the measure of the supplement. The second angle is four times as large as the supplement

or 4x.

Since the angles are supplementary, they add up to 180 degrees.

$x + 4x = 180$

$5x = 180$

$$\frac{5x}{5} = \frac{180}{5}$$

$1x = 36$

$x = 36$

The measure of the supplement is 36 degrees.

4.

Which line or lines are perpendicular to the line y = 6x - 8 ?

a. y = 6x + 1 b. x - 6y = 9

c. x + 6y = -3 d. y = -6x - 1

Two lines are perpendicular if the product of their slopes is equal to -1

The equation of a line is y = mx + b and m is the slope.

For y = 6x – 8, m = 6

$$6\times(\frac{-1}{6}) = (\frac{6}{1})\times(\frac{-1}{6}) = \frac{(6\times-1)}{(1\times6)} = \frac{-6}{6} = -1$$

If an equation is perpendicular to y = 6x – 8, it would have a slope of $\frac{-1}{6}$

A quick observation helps us to eliminate y = 6x + 1 and y = -6y – 1 since the slopes are 6 and -6

Equations x- 6y = 9 and x + 6y = -3 are not in the format y = mx + b.

Let us put them in tha format.

x – 6y = 9

x – 6y + 6y = 9 + 6y

x = 9 + 6y

x – 9 = 9 – 9 + 6y

x – 9 = 6y

$$\frac{x}{6} - \frac{9}{6} = \frac{6y}{6}$$

$$\frac{1x}{6} - \frac{9}{6} = y$$

$(\frac{1}{6})x - \frac{9}{6} = y$

The slope for x – 6y = 9 is $\frac{1}{6}$ This one does not work either.

Let us check the last one

x + 6y = -3

x – x + 6y = -3 - x

0 + 6y = -3 - x

6y = -3 – x

6y = -3 + -x

$\frac{6y}{6} = \frac{-3}{6} + \frac{-x}{6}$

$y = \frac{-3}{6} - + \frac{-1x}{6}$

$y = \frac{-3}{6} + (\frac{-1}{6})x$

The slope for x + 6y = -3 is $\frac{-1}{6}$

Therefore, the line that is perpendicular to y = 6x – 8 is x + 6y = -3

5. Find two angles that are supplementary and vertical at the same time

Two angles are supplementary if they add up to 180 degrees.

When two angles are vertical, they are equal.

Let x be the first angle and let y be the second angle.

x + y = 180

However, x = y since the angles are vertical

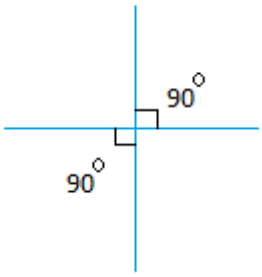
x + x = 180

2x = 180

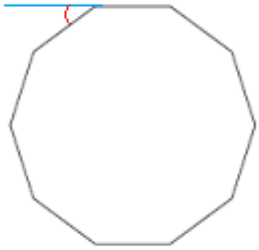
$$\frac{2x}{2} = \frac{180}{2}$$

x = 90 degrees.

The measure of two angles that are vertical and complementary at the same time is 90 degrees.



6. What is the measure of an exterior angle of a regular 10-gon?



One such exterior angle is shown above. There are a total of 10 exterior angles and they are all equal.

Furthermore, the sum of all 10 angles always add up to 360 degrees

Let x be one angle

x + x + x + x + x + x + x + x + x +x = 360

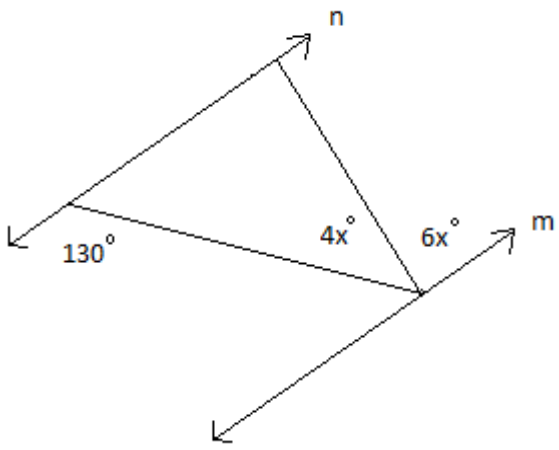
10x = 360

$$\frac{10x}{10} = \frac{360}{10}$$

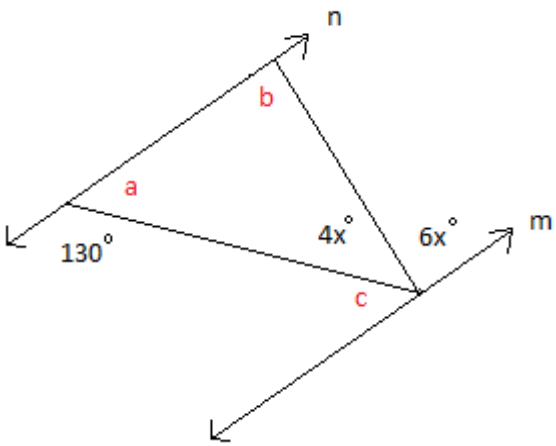
x = 36 degrees

7.

In the figure below, line n is parallel to line m. Find the measure of all angles inside line n and line m.



Label 3 of the missing angles a, b, and c.



a + 130 = 180 (Supplementary angles)

a + 130 – 130 = 180 – 130

a = 50 degrees

b = 6x (alternate interior angles are equal)

50 + 6x + 4x = 180 (angles in a tiriangle add up to 180)

50 + 10x = 180

50 - 50 + 10x = 180 – 50

10x = 130

$$\frac{10x}{10} = \frac{130}{10}$$

x = 13

The angle marked 4x = 4×13 = 52 degrees

The angle marked 6x = 6×13 = 78 degrees

c + 52 + 78 = 180 (3 angles that make a straight line equal 180 degrees)

c + 130 = 180

c + 130 – 130 = 180 – 130

c = 50 degrees

8.

Find the volume and surface area of a sphere with radius 4 cm

Volume = $\frac{4}{3} \times \pi \times r^3$

Volume = $\frac{4}{3} \times 3.14 \times 4^3$

Volume = $\frac{4}{3} \times 3.14 \times 4 \times 4 \times 4$

Volume = $\frac{(4 \times 3.14 \times 4 \times 4 \times 4)}{3}$

Volume = $\frac{803.84}{3}$

Volume = 268 cm^3

Surface area = $4 \times \pi \times r^2$

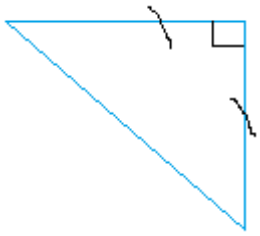
Surface area = $4 \times 3.14 \times 4^2$

Surface area = $4 \times 3.14 \times 4 \times 4$

Surface area = 200.96 cm^2

9. What kind of triangle is the one below? Find all 3 angles.

a. Equilareral b. Right c. Right isosceles d. Scalene



This triangle is a right isosceles triangle. It has a right angle and 2 equal sides.

Let x b one of the base angles

90 + x + x = 180

90 + 2x = 180

90 – 90 + 2x = 180 - 90

$2x = 90$

$$\frac{2x}{2} = \frac{90}{2}$$

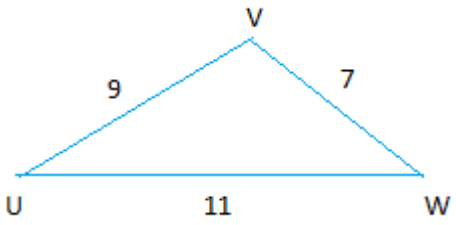
$x = 45$ degrees

10.

In triangle UVW, UV = 9, VW = 7, and UW = 11. Which angle is the biggest?

a. Angle U b. Angle V c. Angle W d. not enough information

The first step is to draw the triangle



Then, the trick is to see that the biggest angle opens up to the biggest side.

The biggest angle is v

11. The surface area of sphere A is 4 times bigger than the area of sphere B. How big is the volume of sphere A compared to sphere B?

The surface area of sphere A is 4 times the surface area of sphere B if the radius of sphere A is 2 times bigger than the radius of sphere B

SA = $4\pi r^2$ (Surface area of sphere B)

Let us double the radius now: 2r

SA = $4\pi(2r)^2$

SA = $4\pi \times 4r^2$

SA = $4 \times (4\pi r^2)$ (Surface area of sphere A)

V = $\frac{4}{3} \times \pi \times r^3$ (Volume of sphere B)

Let us double the radius now: 2r

V = $\frac{4}{3} \times \pi \times r^3$

V = $\frac{4}{3} \times \pi \times (2r)^3$

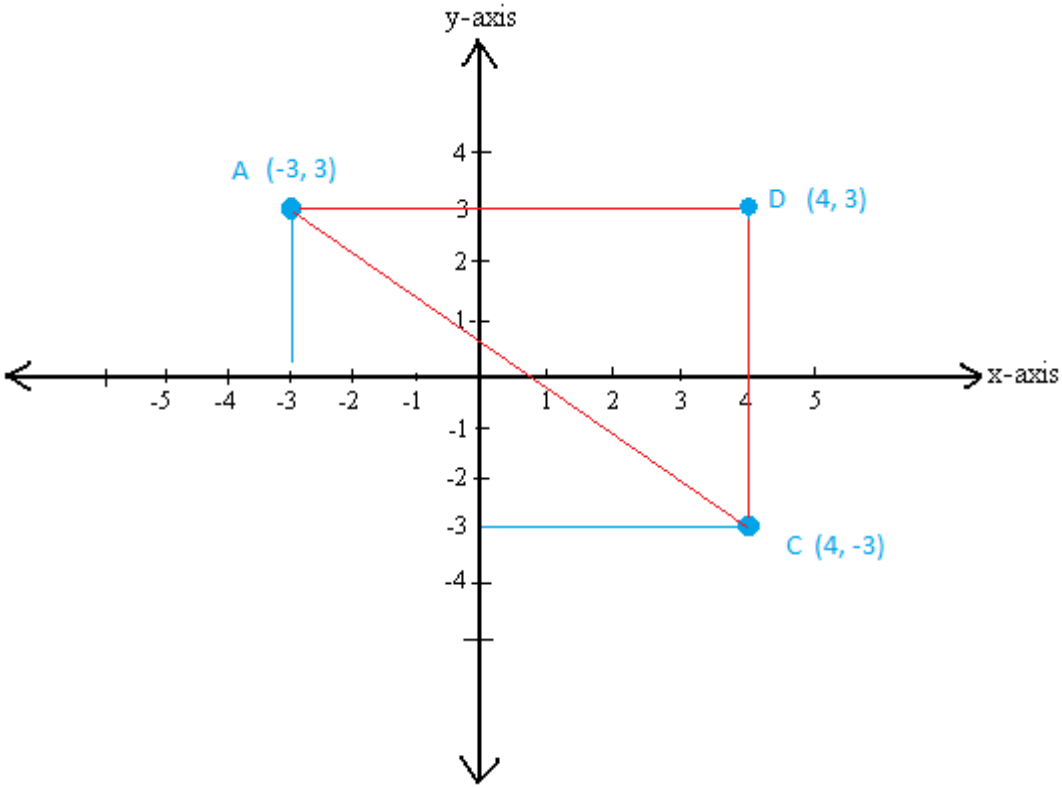
V = $\frac{4}{3} \times \pi \times 8r^3$

V = $8 \times (\frac{4}{3} \times \pi \times r^3)$ (Volume of sphere A)

As you can see the volume of sphere B is multiplied by 8

The volume of sphere A will be 8 times bigger.

12. Graph poins A (-3, 3), C(4, -3), and D(4, 3). Name the figure and show that segment AD is perpendicular to segment CD. Write equations for all 3 segments.



To show that AD is perpendicular to CD, we must find the slopes of AD and CD and then show that the product of the slopes is -1.

Alternatively, we can show that one line is horizontal while the other is vertical. A line is vertical when the slope is undefined. A line is horizontal when the slope is zero

Slope of DC

$$m=\frac{y_1-y_2}{x_1-x_2}$$

m = $\frac{3--3}{4-4}$

m = $\frac{6}{0}$

m is undefined because $\frac{6}{0}$ has no answer

Line DC is vertical

Slope of AD

$$m=\frac{y_1-y_2}{x_1-x_2}$$

m = $\frac{3-3}{4--3}$

m = $\frac{0}{7}$

m = 0

Line AD is horizontal.

Therefore, line AD and DC are perpendicular.

The equation that we are looking for the lines has the format y = mx + b

For line AD, m = 0 , so y = 0x + b. To get b, choose a point on AD such as (4,3)

and replace x = 4 and y = 3 into y = 0x + b

3 = 0×4 + b

3 = 0 + b

b = 3

For line AD, the equation is y = 0x + 3 or y = 3

For line DC, it is not possible to find m. A good observation helps us to see though that x = 4 no matter what y is equal to.

The equation for DC is x = 4

Line AC

Use the points (-3, 3) and (4, - 3) to get the slope

$$m = \frac{-3-3}{4--3} = \frac{-6}{7}$$

y = mx + b = $\frac{-6}{7}$ x + b

Find b now using (-3, 3) on AC

y = $\frac{-6}{7}$ x + b

$3 = \frac{-6}{7} \times -3 + b$

$3 = \frac{18}{7} + b$

$3 - \frac{18}{7} = \frac{18}{7} - \frac{18}{7} + b$

$3 - \frac{18}{7} = b$

$\frac{3}{1} - \frac{18}{7} = b$

$\frac{3}{1} \times \frac{7}{7} - \frac{18}{7} = b$

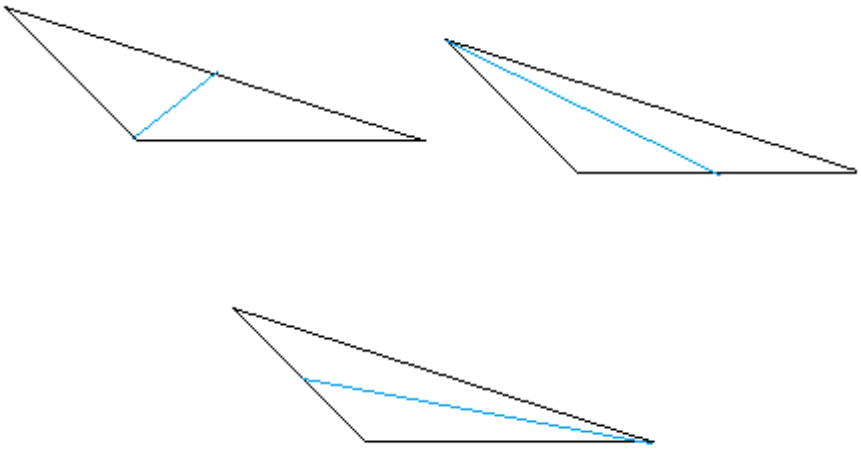
$\frac{21}{7} - \frac{18}{7} = b$

$\frac{3}{7} = b$

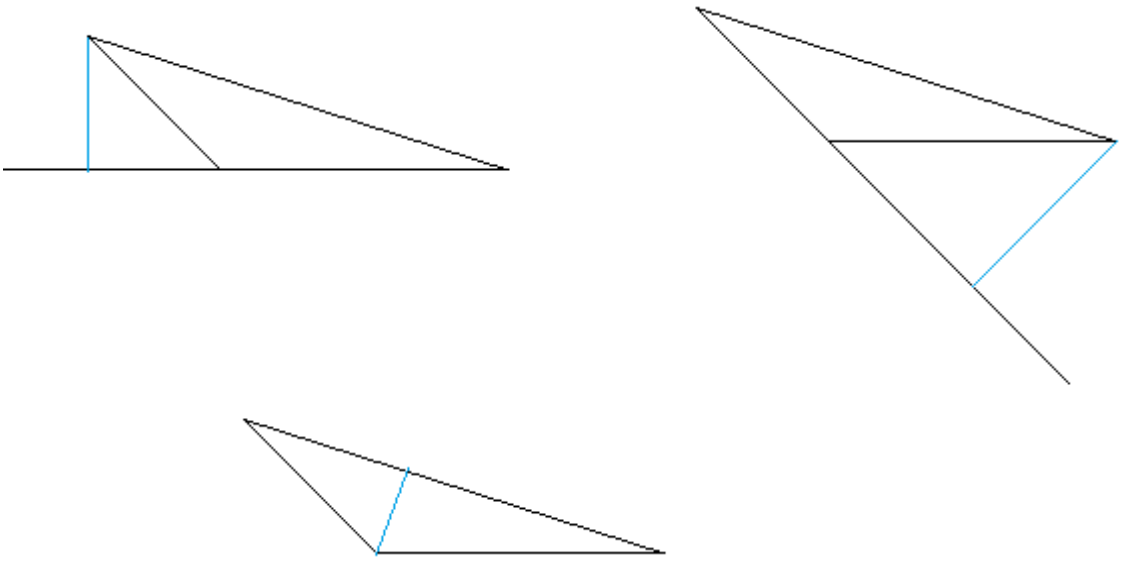
$y = \frac{-6}{7}x + \frac{3}{7}$

13. Show all medians and all heights of this triangle. No construction with straightedge and compass is needed!

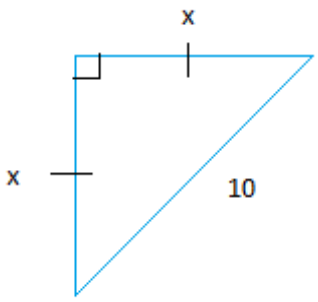
The medians are shown below. It is a line drawn from a vertex to the midpoint of the line that is opposite to the vertex



The heights are shown below in blue.



14. The length of the hypotenuse of an isosceles right triangle is 10 inches. What is the length of one leg?



$c^2 = a^2 + b^2$

c = 10 and a = b = x

$10^2 = x^2 + x^2$

$10^2 = 2x^2$

100 = $2x^2$

$\frac{100}{2} = \frac{2x^2}{2}$

50 = x^2

x = $\sqrt{50}$

x = 7.07 inches

15. Find the volume and surface area of a rectangular prism 3 cm by 5 cm by 10 cm.

Volume = $length \times width \times height$

= $3 \times 5 \times 10$

= 3×50

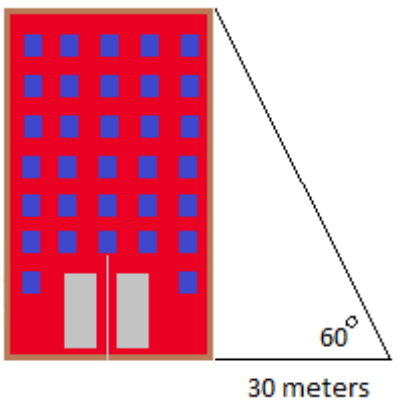
= 150 cm^3

Surface area = $2 \times length \times width + 2 \times length \times height + 2 \times width \times height$

Surface area = $2 \times 3 \times 5 + 2 \times 3 \times 10 + 2 \times 5 \times 10$

Surface area = 30 + 60 + 100 = 190 cm^2

16.



a. Use trigonometric ratio to find the height of this building to the nearest tenth.

let h be the height of the building

$\tan(60) = \frac{h}{30}$

$1.73 = \frac{h}{30}$

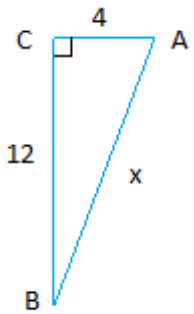
$1.73 \times 30 = \frac{h}{30} \times 30$

51.9 = h

The height of the building is 51.9 meters

b. Which expression will yield the biggest value?

a. sin A b. cos A C. tan A D. tan B



sin A = $\frac{12}{x}$

cos A = $\frac{4}{x}$

tan A = $\frac{12}{4}$

tan B = $\frac{4}{12}$

tan A is bigger than tan B .

sin A is bigger than cos A

We just need to compare tan A with sin A

$x^2 = 12^2 + 4^2$

$x^2 = 144 + 16 = 160$

x = $\sqrt{160}$ = 12.64

sin A = $\frac{12}{12.64}$ = 0.94

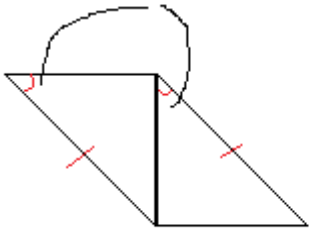
tan A = $\frac{12}{4}$ = 3

tan A has the biggest value

17.

Tell which congruence postulate can be used to prove congruence. Write down SSS, SAS, ASA, or AAS beneath each triangle. If you cannot prove congruence, write N/A.

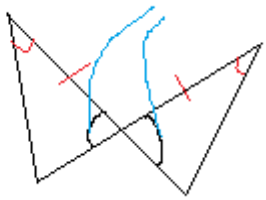
Different angles



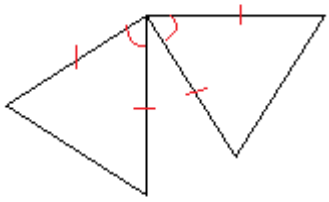
N/A

It appears that SAS would work. However, the included angle is not the same for both triangle

Vertical angles

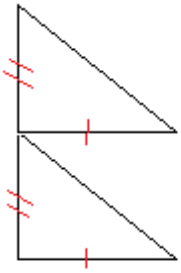


ASA works. Notice that the side is between the vertical angle and the angle in red



SAS

A rotation of 90 degrees of either triangle will help you see better.



N/A

It appears to be SAS, but there is no indication that the angle that looks like a right angle is indeed a right angle.

18.

Write an equation of a line parallel and perpendicular to y = 12x - 4 and passing through (1, 0)

The equation that is parallel to y = 12x – 4 has the same slope as 12x – 4

The equation we are looking for has the following format.

y = mx + b

Since the slope is the same, y = 12x + b

Replace x = 1 and y = 0 into y = 12x + b

0 = 12 times 1 + b

0 = 12 + b

0 – 12 = 12 – 12 + b

-12 = b

The equation that is parallel to y = 12x - 4 is y = 12x – 12

The equation that is perpendicular to y = 12x – 4 has a slope that is the negative reciprocal of 12. In other words, their product is equal to -1

The reciprocal is 12 is $\frac{1}{12}$

Te negative reciprocal is $\frac{-1}{12}$

y = $\frac{-1}{12}x$ + b

Replace x = 1 and y = 0 into y = $\frac{-1}{12}x$ + b

0 = $\frac{-1}{12}\times 1$ + b

0 = $\frac{-1}{12}$ + b

0 + $\frac{1}{12}$ = $\frac{-1}{12}$ + $\frac{1}{12}$ + b

$\frac{1}{12}$ = b

The equation that is perpendicular to y = 12x – 4 is y = $\frac{-1}{12}x$ + $\frac{1}{12}$

19.

What is the standard equation of a circle with center (3, -4) and radius 2?

a. $(x-3)^2+(y-2)^2$ = 4

b. $(x-3)^2+(y+4)^2$ = 4

c. $(x-3)^2+(y-3)^2$ = 4

d. $(x-3)^2+(y+4)^2$ = 4

The standard equation of a circle is $(x-h)^2+(y-k)^2$ = r^2

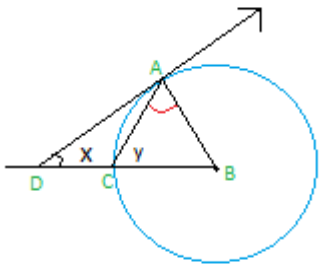
Since r = 2, r^2 = 4

h = 3, so x – h = x – 3

k = -4, so x – k = x - - 4 = x + 4

The correct equation is $(x-3)^2+(y+4)^2$ = 4

20.



If the angle in red is equal to 50 degrees, what is the measure of angle x?

Angle in red is angle CAB, so angle CAB = 50 degrees

Since triangle ABC IS isosceles with AB = CB, angle y or angle ACB is also equal to 50 degrees

angle y + angle ACD = 180 degrees

50 + angle ACD = 180

angle ACD = 130 degrees since 50 + 130 = 180

angle DAB = 90 degrees (tangent to a circle is perpendicular to the radius drawn to the point of tangency)

angle DAC + angle CAB = 90

angle DAC + 50 = 90

angle DAC = 40 since 40 + 50 = 90

angle x + angle DAC + angle ACD = 180

angle x + 40 + 130 = 180

angle x + 170 = 180

angle x = 10 degrees since 170 + 10 = 180

21.

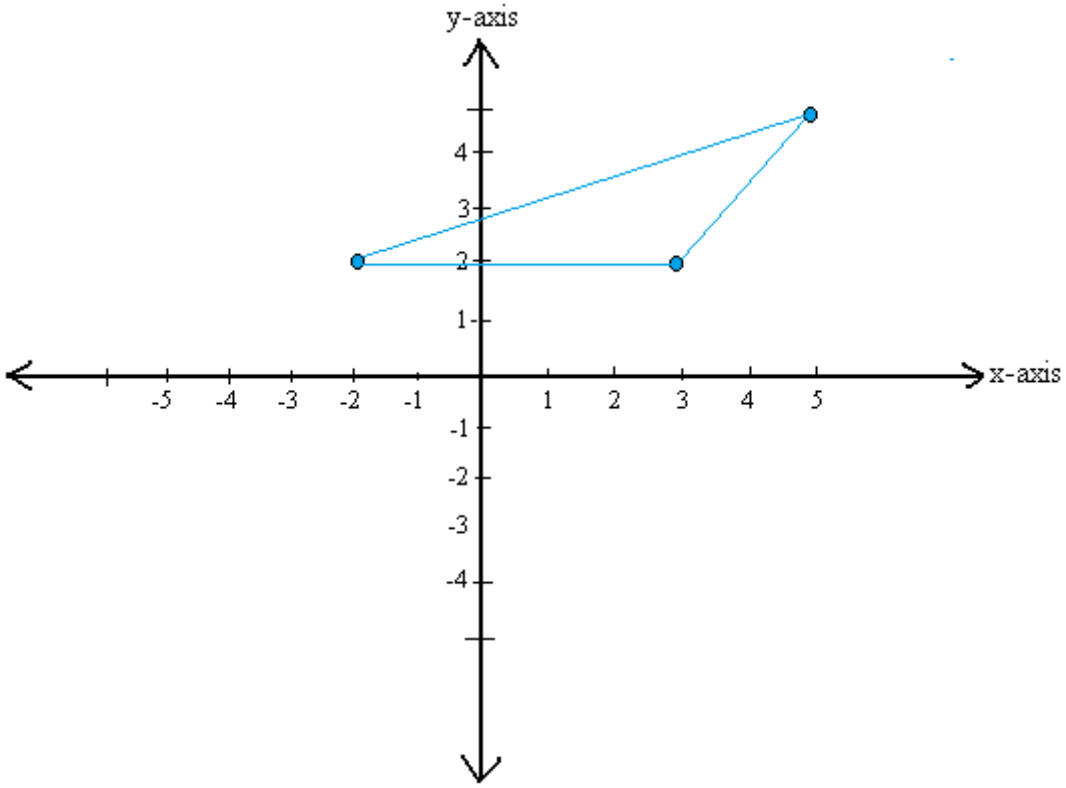
Graph the triangle X(-2,2), Y(3,2), and Z(5,5)

a. Find the image of triangle XYZ under the translation (x - 3, y - 4)

b. Reflect triangle XYZ across the x-axis.

c. Rotage triangle XYZ 270 degrees counterclockwise about X

d. Dilate triangle XYZ by a factor of 2



a.

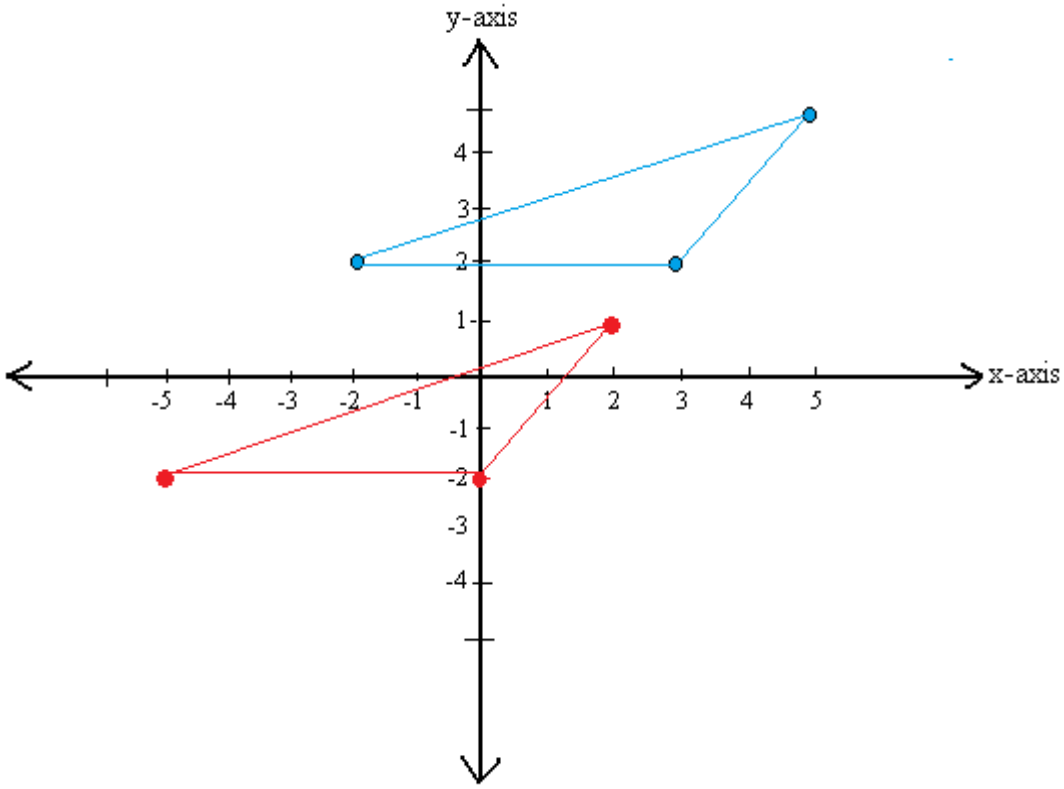
X(-2,2), Y(3,2), and Z(5,5)

The translation of (x-3, y-4) gives these new points

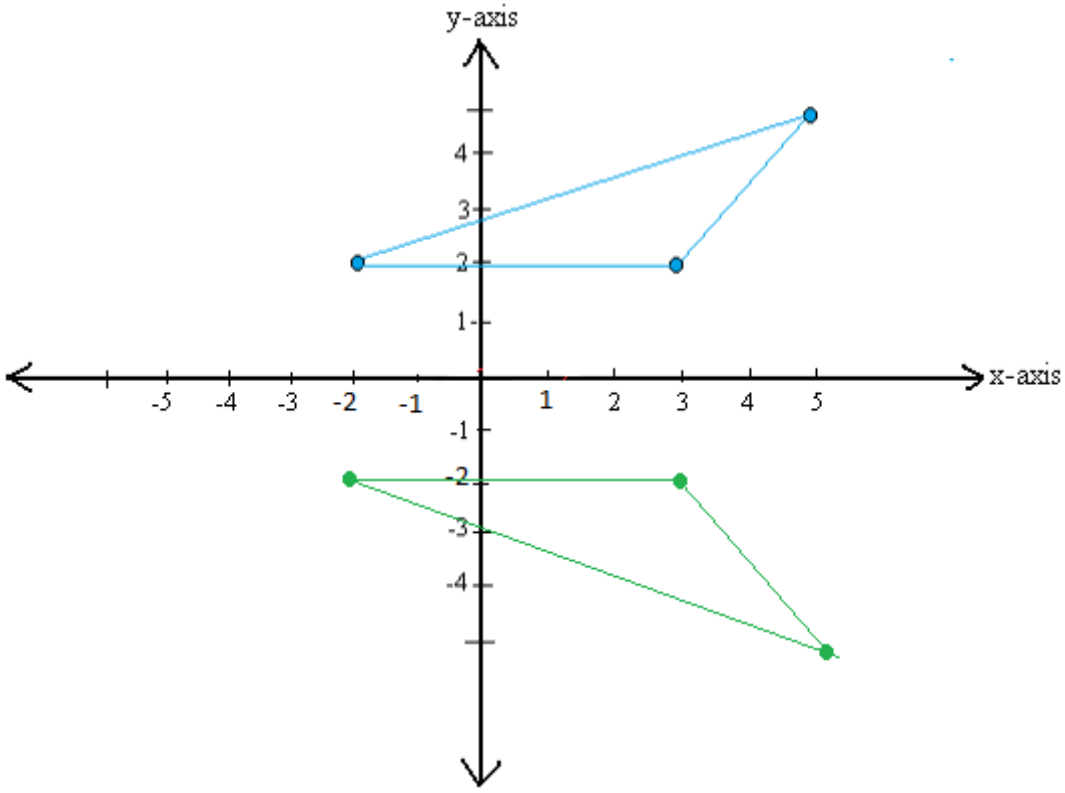
X1(-2-3, 2-4), Y1(3 -3, 2 -4), and Z1(5 – 3, 5 – 4)

Or X1(-5, -2), Y1(0, -2), and Z1(2,1)

The new image is shown below in red



b. The reflection over the x-axis is show below



c.

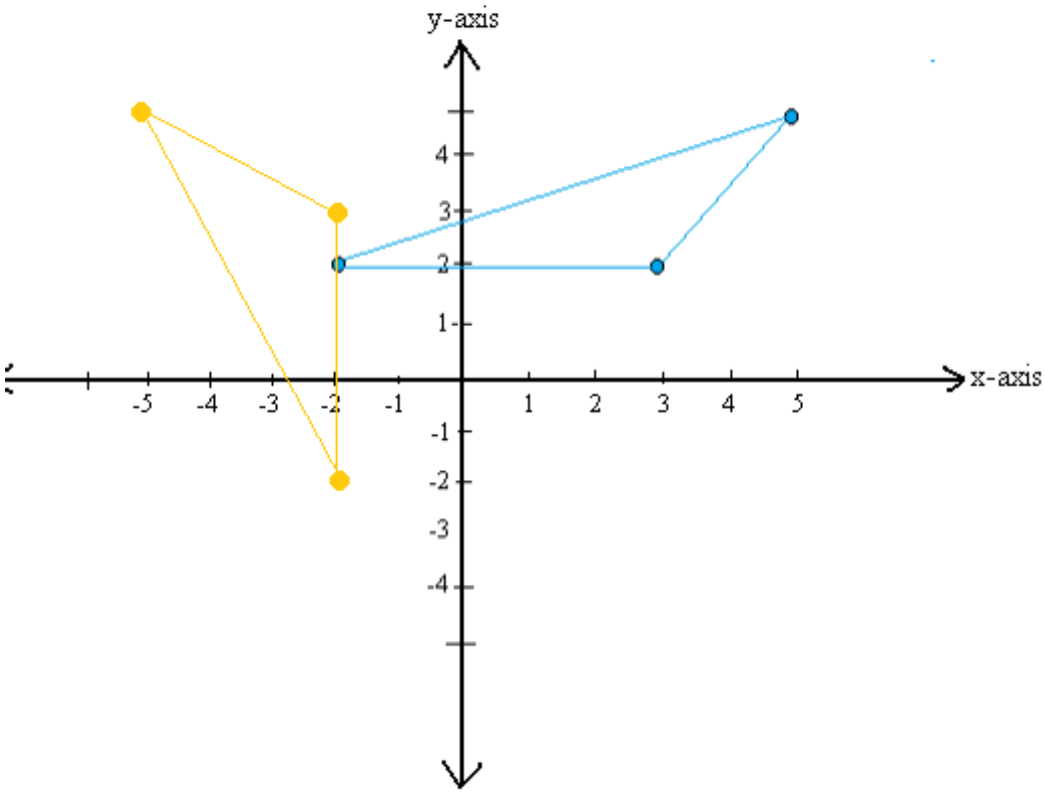
A rotation of 270 degrees clockwise is the same as a rotation of 90 degrees counterclockwise

When you rotate 90 degrees about the **origin** counterclockwise, use this rule.

(a, b) becomes (-b, a)

So (-2,2) becomes (-2, -2), (3,2) becomes (-2, 3), and (5, 5) becomes (-5, 5)

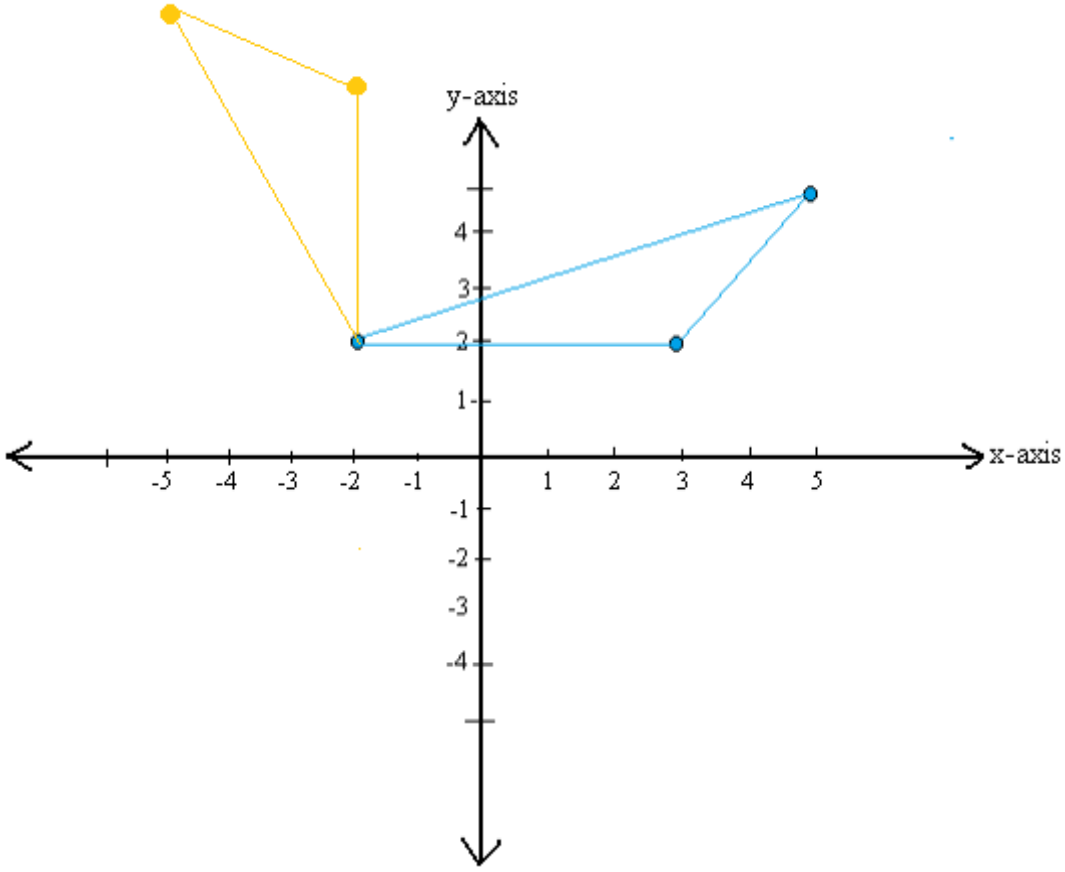
The figure will look like this



Notice though that the point of rotation(-2, 2) is all the way down at (-2, -2).

All you have to do is to move the points of rotation and the other 2 points up 4 units.

The rotation is shown below.



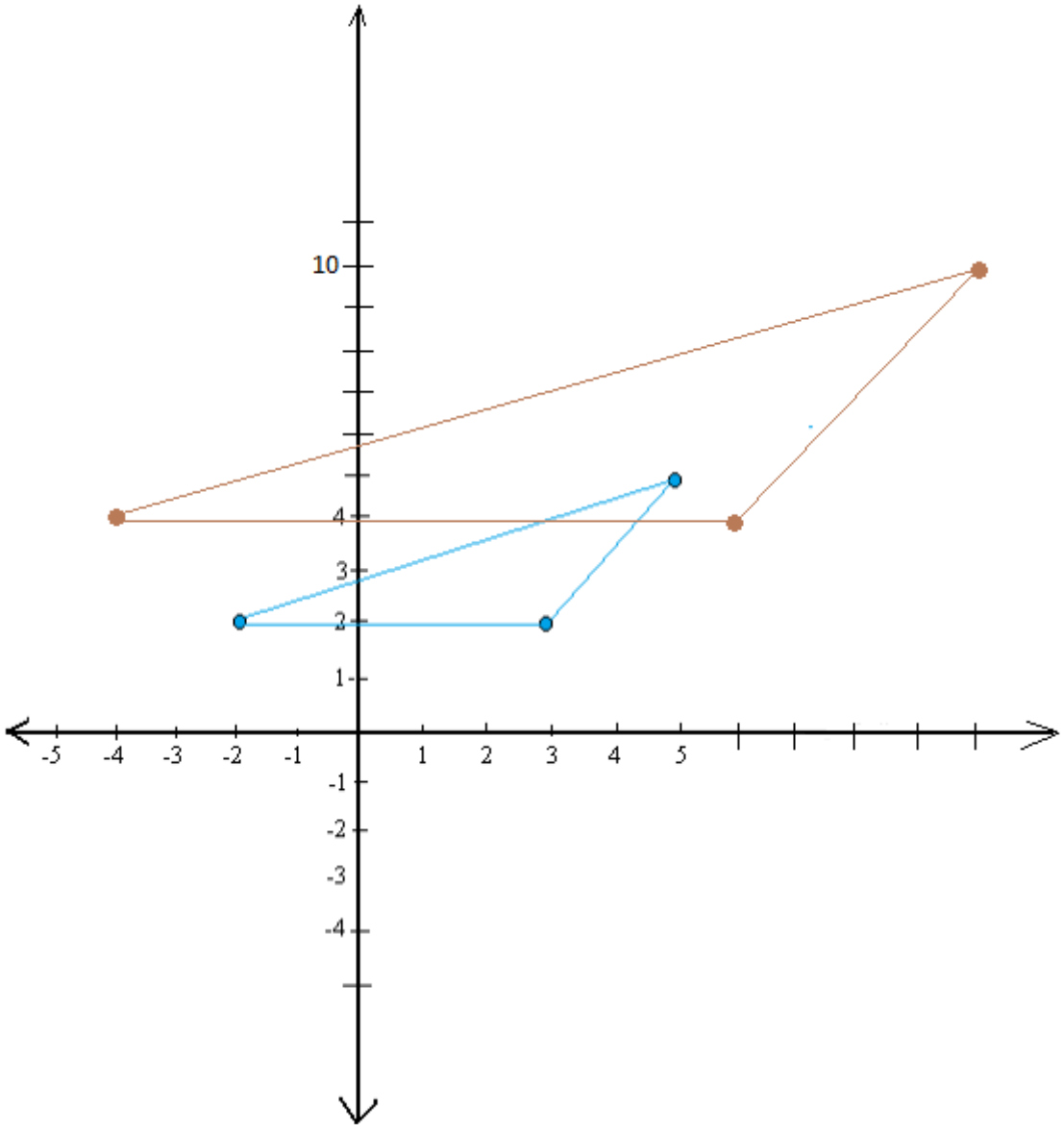
To dilate the triangle X(-2,2), Y(3,2), and Z(5,5) by a factor of 2, just multiply each of the points by 2.

(-2, 2) be comes (-4, 4)

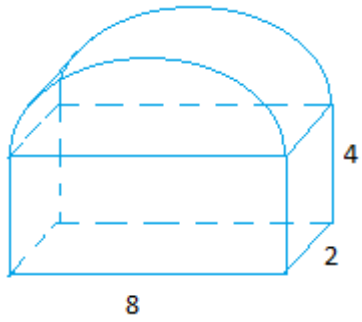
(3, 2) becomes (6, 4)

(5, 5) becomes (10, 10)

Just plot the new points. The dilation is shown below.



22. What is the volume of the figure below?



The shape above is made up of two shapes. It has a rectangle and half a cylinder.

The volume of the rectangle = $length \times width \times height$

= $8 \times 2 \times 4$

= 64

The formula to get the volume of a cylnder is V = $\frac{4}{3}\pi r^3$

r = $\frac{8}{2}$ = 4

V = $\frac{4}{3}\pi r^3$

V = $\frac{4}{3} \times 3.14 \times 4^3$

V = $\frac{4}{3} \times 3.14 \times 4 \times 4 \times 4$

$V = \frac{4}{3} \times 3.14 \times 64$

$V = \frac{4 \times 64 \times 3.14}{3}$

$V = \frac{4 \times 64 \times 3.14}{3}$

$V = \frac{256 \times 3.14}{3}$

$V = \frac{256 \times 3.14}{3}$

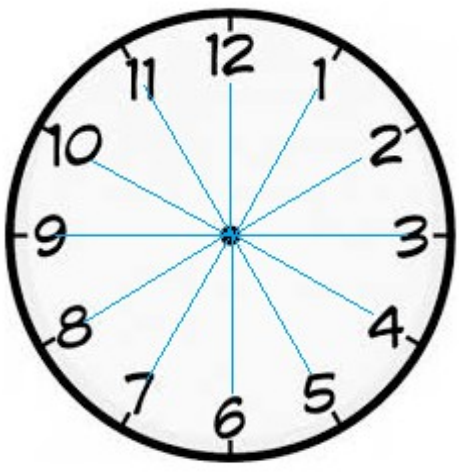
$V = \frac{803.84}{3}$

$V = 268$

Total volume = 268 + 64 = 332

23.

If the clock says 4:00 PM, what is the measure of the angle formed by the hands?



A clock like the one you see above is a circle.

A full turn around a circle is equal to 360 degrees

The circle has also 12 central angles of the same size.

Therefore, each angle is equal to $\frac{360}{12}$ = 30

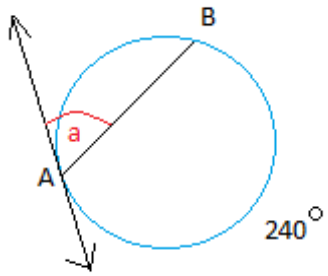
Between the 12 and the 4, there are 4 central angles.

4 times 30 = 120

The angle formed by the 12 hands is 120 degrees

24.

Find the measure of angle a



Theorem: The measure of an angle formed by a tangent and a chord is half the measure of the intercepted arc.

Angle a is formed like that.

Thus, the measure of angle a is half measure of minor arc AB

measure of minor arc AB = 360 – measure of major arc AB

measure of minor arc AB = 360 – 240

measure of minor arc AB = 120 degrees

measure of angle a = half measure of minor arc AB = $\frac{120}{2}$

measure of angle a = 60 degrees

25.

Write an equation of the circle with center (-4,5) and passing through the point (-2, 5)

The standard equation of a circle is $(x-h)^2+(y-k)^2 = r^2$

We know the center since h = -4 and k = 5. However, we must find r.

r is the distance between (-4, 5) and (-2, 5)

We can use the distance formula.

d = $\sqrt{(-4--2)^2+(5-5)^2}$

d = $\sqrt{(-4+2)^2+0}$

d = $\sqrt{(-2)^2}$

d = $\sqrt{4}$

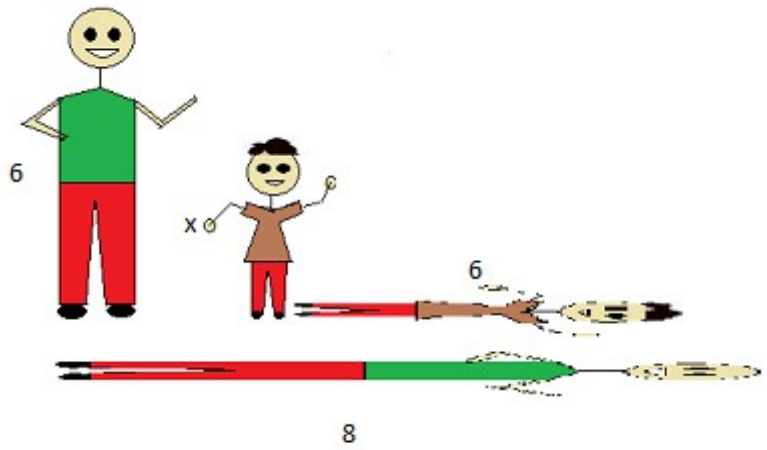
d = 2

The equation becomes $(x--4)^2+(y-5)^2 = 2^2$

$(x+4)^2+(y-5)^2 = 4$

26.

A man is 6 feet tall. When his shadow is 8 feet tall, the shadow of his son standing next to him is only 6 feet. Make a drawing and find out how tall the son is in feet and inches.



Set up a proportion

$$\frac{6}{x}=\frac{8}{6}$$

Cross multiply

$$x\times 8=6\times 6$$

8x = 36

Divide both sides by 8

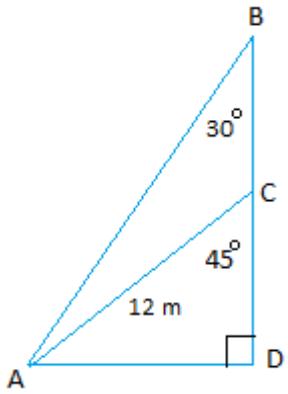
$$\frac{8x}{8}=\frac{36}{8}$$

x = 4.5.

The son is 4.5 feet tall

27.

Use the triangle below to find the lengths of AB and CD



Sin 45 degrees = $\frac{AD}{12}$

0.707 = $\frac{AD}{12}$

Multiply both sides by 12

$0.707 \times 12 = \frac{AD}{12} \times 12$

8.48 = AD

Sin 30 degrees = $\frac{AD}{AB}$

$0.5 = \frac{8.48}{AB}$

Multiply both sides by AB

$0.5 \times AB = \frac{8.48}{AB} \times AB$

$0.5 \times AB = 8.48$

Divide both sides by 0.5

$\frac{0.5\,AB}{0.5} = \frac{8.48}{0.5}$

AB = 16.96

Angle CAD must equal to 45 degrees since 45 + 45 + 90 = 180 degrees.

Triangle ADC is isosceles. This means that AD = CD = 8.48

28.

When the length of 1 side of hexagon is 3 meters, the area is 23.38 square meters. What is the area if the hexagon has a side length of 6 meters?

Let A be the area with a side length of 3 meters

Let A1 be the area with a side length of 6 meters

$$\frac{A}{3^2}=\frac{A1}{6^2}$$

Notice the exponent of 2. That is how it works when doing ratio of area to side length

$$\frac{23.38}{9}=\frac{A1}{36}$$

Cross multiply

$$23.38\times 36=9\times A1$$

841.68 = 9A1

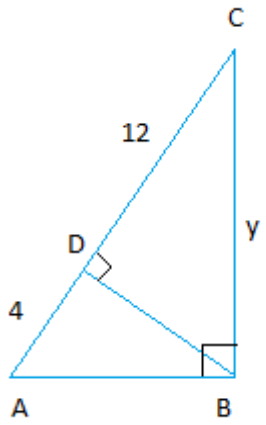
Divide both sides by 9

$$\frac{841.68}{9}=\frac{9\text{A1}}{9}$$

93.52 = A1

29.

Use the triangle below to find y



All 3 triangles are similar, so you can use ratios of corresponding sides to solve for y

Let DC = Longer leg of triangle CDB

Let CB = Longer leg of triangle CBA

Let CB also equal the hypotenuse of triangle CDB

Let CA equal the hypotenuse of triangle CBA

$$\frac{DC}{CB}=\frac{CB}{CA}$$

$$\frac{12}{y}=\frac{y}{16}$$

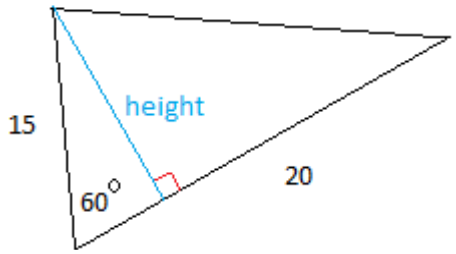
$$y\times y=12\times 16$$

y^2 = 192

$y=\sqrt{192}$ = 13.9

30.

Two sides of a triangle measure 15 and 20. If the angles between these sides is 60 degrees, find the area of the triangle.



Area = $\frac{20 \times height}{2}$

Sin 60 degrees = $\frac{height}{15}$

0.86 = $\frac{height}{15}$

Multiply both sides by 15

$0.86 \times 15 = \frac{height}{15} \times 15$

$0.86 \times 15 = height$

12.9 = height

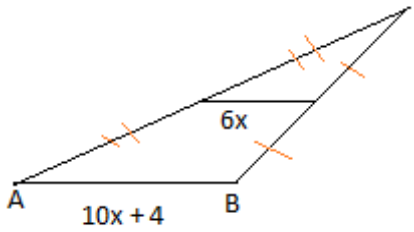
$$Area = \frac{20 \times 12.9}{2}$$

$$Area = \frac{258}{2}$$

Area = 129

31.

Use the triangle below to find x



The important thing to remember is that the line in the middle of the triangle is half line AB

$$6x = \frac{10x + 4}{2}$$

Multiply both sides by 2

$$2 \text{ times } 6x = \frac{10x + 4}{2} \times 2$$

$12x = 10x + 4$

$12x - 10x = 10x - 10x + 4$

$2x = 4$

$$\frac{2x}{2} = \frac{4}{2}$$

$x = 2$

32.

Which statement will turn a parallelogram into a rhombus?

a. The diagonals bisect each other.

b. Opposite angles are equal

c. 4 equal sides

d. Opposite sides are equal

e. The diagonals are perpendicular

The answer is c and e.

33.

The volume of a cone is one-third that of the cylinder. Try to explain why this is the case **without** doing any math computation.

Just use a cylinder and a cone that have the same base and height.

If you fill the cone with sands, it will take 3 cones to fill the cylinder

This means that the volume of the cylinder is 3 times the volume of the cone

Therefore, the volume of the cone is one-third the volume of the cylinder

34.

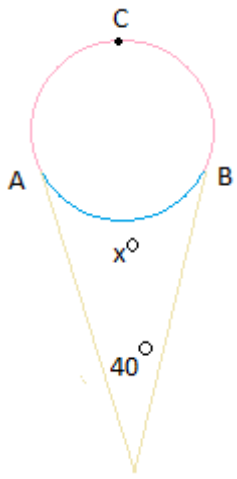
For the shape below

a. Find the angle x

b. Suppose that the circle represents 1 scoop of ice cream and it weighs 4 ounces.

Estimate how much of the ice cream is inside the cone

Hint: The portion that is inside is shown with a blue arc.



Notice that the minor is in blue and the major arc is in red.

Let x be the measure of minor arc AB. Then, measure of major arc AB is 360 -x

Theorem: The angle of 40 degrees made by the intersect outside the circle is

half (measure of arc in red – measure of arc in blue)

$40 = \frac{[360 - x] - x}{2}$

$40 = \frac{360 - 2x}{2}$

40 = 180 – x

x = 140 degrees since 180 – 140 = 40

b.

$\frac{140}{360} = \frac{14}{36} = \frac{7}{18}$

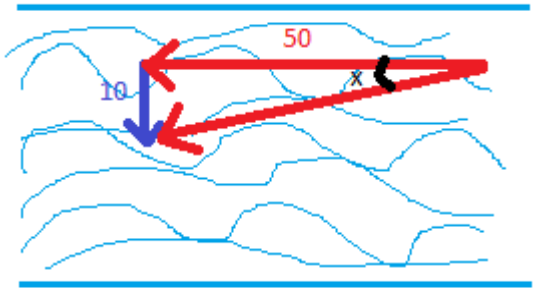
The fraction that is inside is 7 over 18

$\frac{7}{18}$ of 4 ounces = $\frac{7}{18} \times 4$ = $\frac{7}{18} \times \frac{4}{1}$ = $\frac{28}{18}$ = 1.55

About 1.55 ounces of ice cream is inside the cone

35.

A boat is traveling west on a river with a speed of 50 miles per hour. The river flows directly south with a speed of 10 miles per hour. What is the boat's resultant speed and direction?



The resultant speed is the hypotenuse of the triangle. Let that be x

$$x^2 = 10^2 + 50^2$$

$$x^2 = 100 + 2500$$

$$x^2 = 100 + 2500$$

$$x^2 = 2600$$

x = $\sqrt{2600}$

x = 50.99 miles per hour

To find the direction, we need to get the measure of the angle x

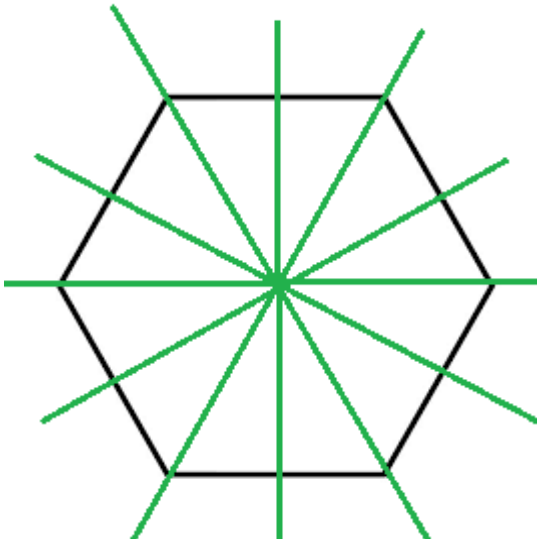
$$\tan x = \frac{10}{50}$$

$\tan x = 0.20$

Using a scientific calculator, $x = 11.31$ degrees

36.

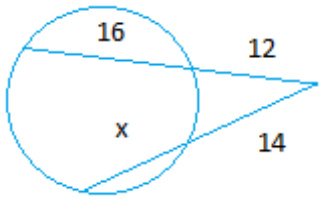
How many lines of symmetry are in a regular hexagon?



A hexagon has 6 lines of symmetry

37.

Use the figure below to find x



The geometric formula to use to solve this problem is

$(16+12)\times 12=(x+14)\times 14$

28×12 = 14x + 196

336 = 14x + 196

336 – 196 = 14x + 196 - 196

140 = 14x

$\frac{140}{14}=\frac{14x}{14}$

10 = x

38.

Use indirect proof to show tht any triangle must have at the most 1 right angle

Indirect proof: Assume the opposite and show that this will lead to a contradiction

Proof:

Assume a triangle has 2 right angles.

This means that the 2 angles add up to 180 degrees

Since the sum of the angle in any triangle add up to 180 degrees, the last angle must be zero.

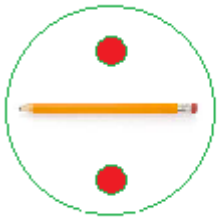
This is a contradiction since there is no such thing as a zero angle

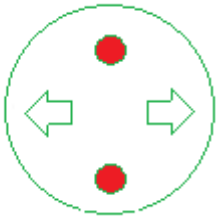
Therefore, assuming that a triangle can have more than 1 right angle is wrong.

All triangles then have at the most 1 right angle

39.

Which figure has rotational symmetry?





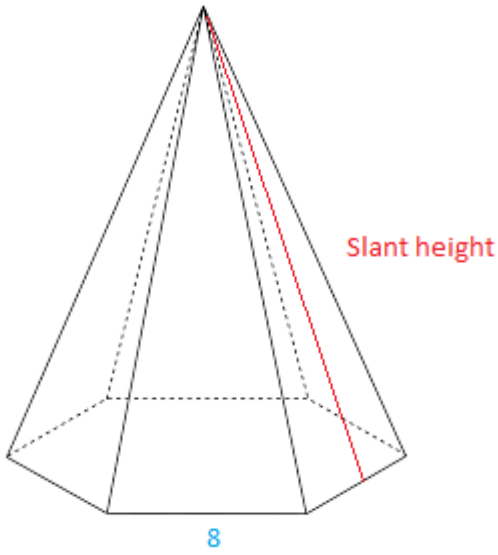
A figure has rotational symmetry if the figure does not change when it is rotated 180 degrees or less

The figure on the left does not have rotational symmetry since it must be ratated 360 degrees to remian the same.

The figure on the right has rotational symmetry since it does not change after it is rotated 180 degrees

40.

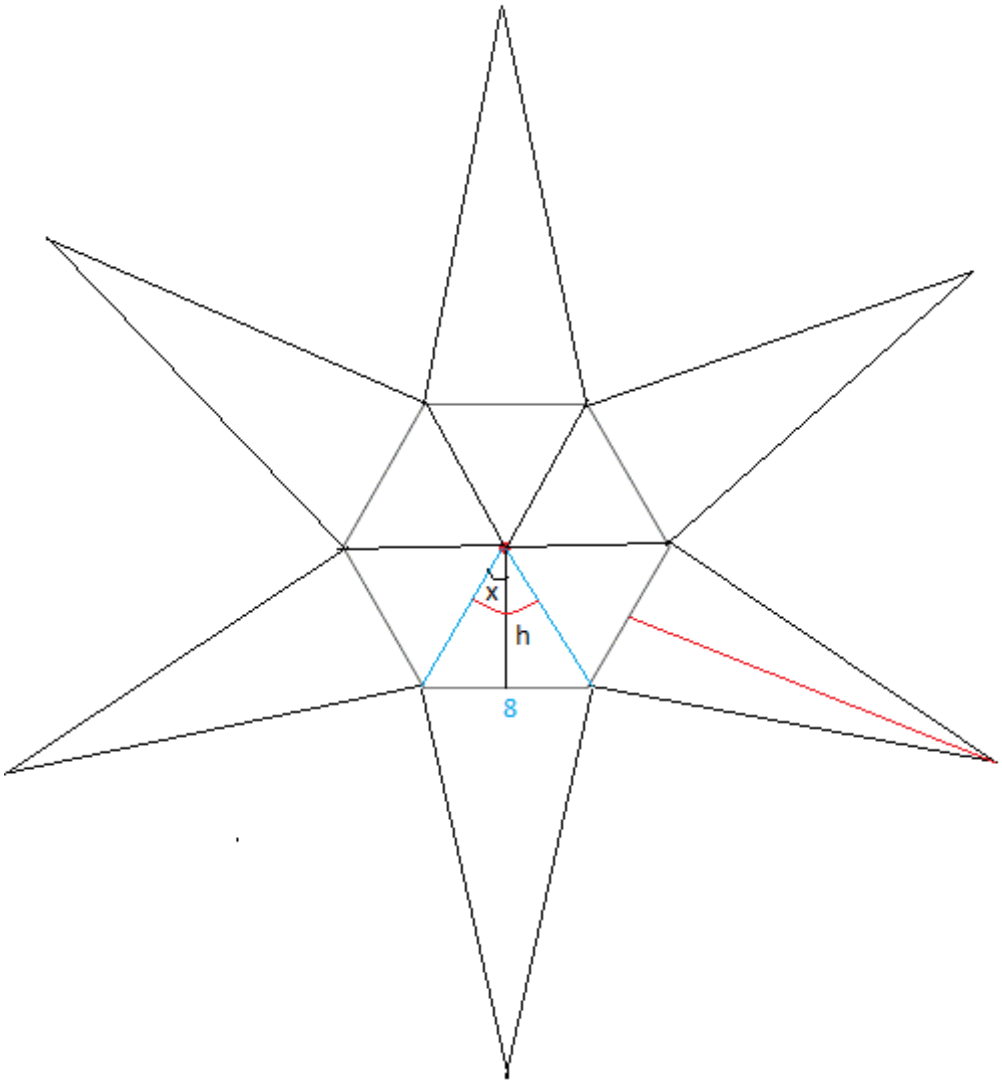
Find the surface area of a hexagonal pyramid if one side of the base is equal to 8 inches and the slant height is 12 inches



The surface area of this pyramid is the sum of the area of the base and the lateral area

The lateral area is the area of the six triangles.

The red line is the slant height of 1 triangle



First, find the area of the base.

We can find the area of the triangle with 2 blue sides and then times that by 6 to get the area of the base.

Area $= \frac{8 \times h}{2}$ h is also called the apothem.

Let us find h by using a trigonometric ratio

$\tan x = \frac{4}{h}$

The angle in red is a central angle. There are 6 such angles and they are all equal.

If we put all six angles together, it makes 360 degrees.

1 angle is equal then to $\frac{360}{6}$ = 60 degrees

$x = \frac{60}{2}$ = 30 degrees

$\tan 30\, degrees = \frac{4}{h}$

$0.577 = \frac{4}{h}$

Multiply both sides by h

$0.58 \times h = \frac{4}{h} \times h$

$0.58 \times h = 4$

Divide both sides by 0.58

$\frac{0.58 \times h}{0.58} = \frac{4}{0.58}$

h = 6.9

$$\text{Area} = \frac{8 \times h}{2} = \frac{8 \times 6.9}{2} = \frac{55.2}{2} = 27.6$$

$$\text{Area of base} = 27.6 \times 6 = 165.6 \quad inches^2$$

Now, let us find the lateral surface.

Again, we just need to find the area of 1 triangle and then times that by 6 to get the lateral surface.

$$\text{Area of 1 of the triangles with a slant height of 12 inches} = \frac{8 \times 12}{2}$$

Area = 96 over 2 = 48

$$\text{Area of all 6 triangles} = 48 \times 6$$

$$= 288 \quad inches^2$$

Total surface area of the whole figure is 288 + 165.6 = 453.6 $inches^2$