9th grade math test solution

1.

Solve each equation separately

$\frac{(3y+1)}{2}$ = 5

Multiply both sides of the equation by 2

$\frac{[2\times(3y+1)]}{2}$ = 5×2

$\frac{2}{2}\times(3y+1)$ = 10

$1\times(3y+1)$ = 10

3y + 1 = 10

3y + 1 – 1 = 10 – 1

3y = 9

$\frac{3}{3}$ y = $\frac{9}{3}$

1y = 3

y = 3

$\frac{4x}{3}$ = 8

Multiply both sides of the equation by 3

$\frac{3\times4x}{3}$ = 8×3

$\frac{3}{3}\times4x$ = 24

$1\times4x$ = 24

4x = 24

$\frac{4}{4}$ x = $\frac{24}{4}$

$1\ x = 6$

$x = 6$

$$\frac{z}{3} + \frac{z}{4} = 2$$

Write both fractions with the same denominator

$$\frac{z}{3} \times \frac{4}{4} + \frac{z}{4} \times \frac{3}{3} = 2$$

$$\frac{4z}{12} + \frac{3z}{12} = 2$$

$$\frac{7z}{12} = 2$$

Multiply both sides of the equation by 12

$$\frac{12 \times 7z}{12} = 2 \times 12$$

$$\frac{12}{12} \times 7z = 24$$

$1 \times 7z = 24$

$7z = 24$

$$\frac{7}{7} z = \frac{24}{7}$$

$$1z = 3\frac{3}{7} = 3.42$$

$z = 3.42$

So $y = 3$, $x = 6$, and $z = 3.42$

y is not bigger than x, so the answer is not A

x is not equal to y, so the answer is not B

y is not bigger than z, so the answer is not C

x is bigger than z, so the answer is D.

2.

A man selling hotdogs, french fries, and a soda as a combo meal at a fairground has to pay the county 250 dollars each day plus 2 dollars per combo meal sold. The price of a combo is 7 dollars How many combo meals must he sell each day to break even?

Break even means that the revenue = cost

In order words, the man does not lose money and he does not make money either

Let x be the number of combo meals he sell in a day

revenue = 7 × x = 7x

Cost = fixed cost plus variable cost = 250 + 2x

7x = 250 + 2x

7x – 2x = 250 + 2x – 2x

5x = 250

Since 5 × 50 = 250, x = 50

The man must sell 50 combo meals to break even.

3.

Simplify $\left(4^{(1/3)}\right)^3 \times \left(6^{(1/6)}\right)^6 \times \left(3^{(1/2)}\right)^4$

$\left(4^{(1/3)}\right)^3 = 4^{(1/3)} \times 4^{(1/3)} \times 4^{(1/3)} = 4^{(1/3+1/3+1/3)} = 4^{(3/3)} = 4^1 = 4$

In general $\left(x^{(1/n)}\right)^n = x$

$\left(6^{(1/6)}\right)^6 = 6$

$3^{(1/2)} \times 3^{(1/2)} \times 3^{(1/2)} \times 3^{(1/2)} = 3^{(4/2)} = 3^2 = 9$

Therefore, $\left(4^{(1/3)}\right)^3 \times \left(6^{(1/6)}\right)^6 \times \left(3^{(1/2)}\right)^4 = 4 \times 6 \times 9 = 24 \times 9 = 216$

4. A and B are independent events.

If P(A) = $\frac{5}{8}$ and P(A and B) = $\frac{1}{4}$, what is P(B)?

If A and B are independent events, P(A and B) = P(A) × P(B)

$$\frac{1}{4} = \frac{5}{9} \times \text{P(B)}$$

$$\text{P(B)} = \frac{(\frac{1}{4})}{(\frac{5}{9})} = \frac{1}{4} \times \frac{9}{5} = \frac{(1 \times 9)}{(4 \times 5)} = \frac{9}{20}$$

5.

a. Choose the correct information about the the graphs of the two lines below.

5x - y - 2 = 0
y - 3x = 9

a. A. Intersect at (5.5, 25.5) B. Parallel C. Identical D. Perpendicular

Write each equation in standard form or y = mx + b with m = slope and b = y-intercept

5x – y – 2 = 0

5x – y + **y** – 2 = 0 + **y** (Add y to both sides)

5x – 2 = y

y = 5x - 2 (Standard form)

y - 3x = 9

y – 3x + 3x = 9 + 3x (Add 3x to both sides)

y = 9 + 3x

y = 3x + 9 (Standard form)

Since the slope are not the same, the lines of these 2 equations will intercept.

Find the point of interception

If the lines intercept, then they must have the same y-value.

$y = 5x - 2$

$y = 3x + 9$

Since they have the same y-value, then $y = 5x - 2 = 3x + 9$

Solve $5x - 2 = 3x + 9$

$5x - 2 + 2 = 3x + 9 + 2$ (Add 2 to both sides)

$5x = 3x + 11$

$5x - 3x = 3x - 3x + 11$ (Subtract 3x from both sides)

$2x = 11$

Since 2 tims 5.5 = 11, x = 5.5

y = 3 times 5.5 + 9

$y = 16.5 + 9 = 25.5$

The answer is A

b. What are the slopes of the graphs?

When the equations are in standard form, the slope is m or the number on the left of x

For $y = 5x - 2$, the number on the left of x is 5, so the slope is 5

For $y = 3x + 9$ the number on the left of x is 3, so the slope is 3

b. ______5_______ and ____3_________

6.

A rectangle has a perimeter of 18 inches. The long side is 1 inch more than three times the small side. How big is the small side?

The small side is ____________

Let x be the short side.

Let y be the long side

$y = 3x + 1$

To get the perimeter, we need to add all 4 sides

$$\begin{aligned}\text{Perimeter} &= x + x + y + y \\ &= x + x + 3x + 1 + 3x + 1 \\ &= 2x + 3x + 1 + 3x + 1 \\ &= 8x + 2\end{aligned}$$

Perimeter = $8x + 2$

$18 = 8x + 2$

Solve $18 = 8x + 2$

$18 - 2 = 8x + 2 - 2$ (Subtract 2 from both sides)

$16 = 8x$

Since $16 = 8 \times 2$, $x = 2$

The short side = 2

The long side $= 3 \times 2 + 1 = 7$

Indeed $2 + 2 + 7 + 7 = 4 + 14 = 18$

7.

The seventh term of the sequence 16, 9 , 2, -5,... is

A. -37 B. 37 C. 26 D. -26

16 – 7 = 9

9 – 7 = 2

2 – 7 = -5

Just subtract 5 to get the next number

-5 – 7 = -12

-12 – 7 = -19

-19 – 7 = -26

The seventh term is -26

8.

Find the degree of $2x^3 + -6x^4 + 4x^2 - 1$

The degree is the term with the highest exponent

The term with the highest exponent is $-6x^4$, so the degree is 4

9.

Find the greatest common factor of the terms of $16x^6 + 8x^4 + 2x^{12}$

A. $8x^{12}$ B. $2x^4$ C. x^6 D. $4x^6$

$16x^6 = 2 \times x^4 \times 8 \times x^2$

$8x^4 = 2 \times x^4 \times 4$

$2x^{12} = 2 \times x^4 \times x^8$

As you can see the greatest common factor is $2 \times x^4$

10.

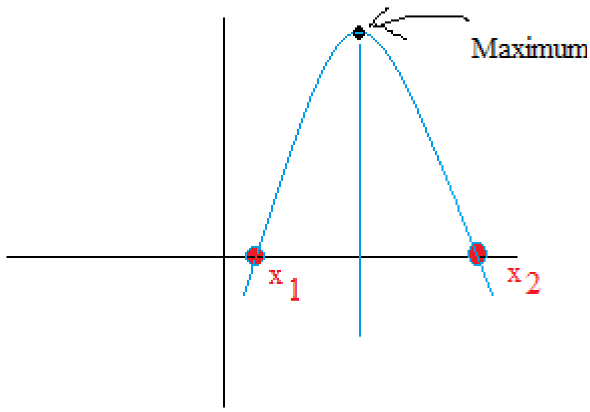
What is the maximum or minimum value of y = x^2 + 3x – 10 ?

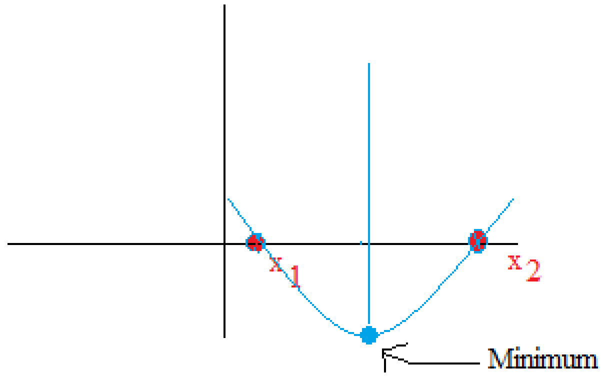
This equation is a parabola. The maximum or minimum value is illustrated below

If the number on the left of x^2 is positive, we get a minimum

If the number on the left of x^2 is negative, we get a maximum

Since there is no negative sign on the left of x^2, we are looking then for a minimum.





As you can see, the x-value of the maximum or minimum is between x_1 and x_2

To find the y, just plug in x into the equation y = x^2 + 3x – 10

To find x_1 and x_2, solve the equation for x.

x^2 + 3x – 10 = 0

(x_1 + blank) × (x_2 + blank) = 0

To find blanks, look for factors of -10 that add up to 3

-10 = 10 × -1

= -10 × 1

= 5 × -2

= -5 × 2

The pair of factors that add up to 3 is 5 and -2 since 5 + -2 = 3

Replace the two blanks with 5 and -2

(x_1 + 5) × (x_2 + -2) = 0

(x_1 + 5) × (x_2 + -2) is zero if (x_1 + 5) = 0 or (x_2 + -2) = 0

x_1 + 5 = 0

x_1 + 5 – 5 = 0 – 5

x_1 = -5

x_2 + -2 = 0

x_2 + -2 + 2 = 0 + 2

x_2 = 2

Since the x-value of the minimum is between x_1 and $x_{2,}$ Just find out what number is between -5 and 2

Look at this list: -5, -4, -3 , -2, -1, 0, 1, 2

-5, -4, -3 , -2, middle number -1, 0, 1, 2

The middle number is between -2 and -1. That number is -1.5

The average will give you the same answer

$$\frac{(-5+2)}{2} = \frac{-3}{2} = -1.5$$

y = x^2 + 3x – 10

y = $(-1.5)^2$ + 3 (-1.5) – 10

y = 2.25 + -4.5 – 10

y = -2.25 – 10

y = -12.25

The minimum value is -12.25

11.

Table 1

x	1	2	3	4
y	3	7	11	15

Table 2

x	1	1	2	4
y	3	7	11	15

A relation is a function if the x-value does not repeat itself.

Look at table 2 and you will see that the 1 repeats itself. Therefore, table 2 is not a function.

In table 1, the x-values are all different, so table 1 is a function.

We can model table 1 with a linear function.

First, find the slope using 2 points. We can use (1, 3) and (2, 7)

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

$$m = \frac{7-3}{2-1} = \frac{4}{1}$$

m = 4

y = mx + b = 4x + b

Find b now using (1, 3)

y = 4x + b

3 = 4 × 1 + b

3 = 4 + b.

Since 3 = 4 + -1, b = -1

y = 4x + -1

f(x) = 4x + -1

12.

Evaluate |y – x| + $-y^2$ + 2z + $x^2 y^3 z^{-2}$ for x = 3, y = 2, and z = -2

|y – x| + $-y^2$ + 2z + $x^2 y^3 z^{-2}$ = | 2 - 3 | + - 2^2 + 2 × -2 + $3^2 2^3 (-2)^{-2}$

= |-1| + -4 + -4 + 9 × 8 × $(-2)^{-2}$

= 1 + -8 + 72 × $(-2)^{-2}$

= -7 + 72 × $\frac{1}{(-2)^2}$

|y – x| + $-y^2$ + 2z + $x^2 y^3 z^{-2}$ = -7 + $\frac{72}{(-2)^2}$

= - 7 + $\frac{72}{4}$

$= -7 + 18$

$= 11$

13.

There are 6 red balls and 4 blue balls in a bag.

Find each probability

P(red and red) with replacing

P(red and blue) with replacing

P(red and red) without replacing

P(red and blue) without replacing

Replacing means that the red ball is put back in the bag so that there are still 6 red balls.

Without replacing means that the red ball is **not put back** in the bag so that there are 5 red balls
in the bag.

Red and red means that a red ball is picked the first time and another red is picked the second time.

Red and blue means that a red ball is picked the first time and a blue ball is picked the second time.

With replacing

P(red and red) = P(red) × P(red) = $\frac{6}{10} \times \frac{6}{10} = \frac{36}{100} = \frac{9}{25}$

P(red and blue) = P(red) × P(bue) = $\frac{6}{10} \times \frac{4}{10} = \frac{24}{100} = \frac{6}{25}$

Without replacing

P(red and red) = P(red) × P(red if red is not replaced) = $\frac{6}{10} \times \frac{5}{9} = \frac{30}{90} = \frac{1}{3}$

P(red and blue) = P(red) × P(bue if red is not repalced) **=** $\frac{6}{10} \times \frac{4}{9}$ **=** $\frac{24}{90}$ **=** $\frac{4}{15}$

14.

Solve 3(2x - 1) = 4x + 5

3 (2x – 1) = 4x + 5

6x – 3 = 4x + 5 (Multiply 3 by 2x and 3 by - 1)

6x – 4x - 3 = 4x - 4x + 5 (Subtract 4x from both sides)

2x – 3 = 5

2x – 3 + 3 = 5 + 3 (Add 3 to both sides)

2x = 8

$\frac{2}{2}x = \frac{8}{2}$ (Divide both sides by 2)

1x = 4

x = 4

15.

Simplify ($4x^3$ - $5x^3$ + x + 8) - ($3x^3$ - 4x + 9 + $6x^2$)

$4x^3$ - $5x^3$ + x + 8 - ($3x^3$ - 4x + 9 + $6x^2$)

= $4x^3$ - $5x^3$ + x + 8 - $3x^3$ + 4x - 9 - $6x^2$ (Remove parentheses)

= - $1x^3$ + x + 8 - $3x^3$ + 4x - 9 - $6x^2$ (Simplify $4x^3 - 5x^3$

= - $1x^3$ - $3x^3$ + x + 4x - 9 + 8 - $6x^2$ (Combine like terms)

= $-4x^3$ + 5x – 1 - $6x^2$

16.

What is the standard form of the product (3x - 1) (5x + 3) ?

A. $15x^2$ + 2x + 3 B. $15x^2$ + 4x - 3 C. $15x^2$ + 4x + 3 D. $15x^2$ + 2x - 3

(3x - 1) (5x + 3) = (3x + -1) (5x + 3)

= 3x times 5x + 3x times 3 + -1 times 5x + -1 times 3

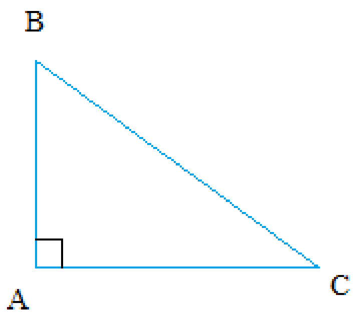
= $15x^2$ + 9x + -5x + -3

= $15x^2$ + 4x + -3

The answer is B.

17.

Triangle ABC is a right triangle with a right angle at A. Which of the following is false ?



The false trigonometric ratio is sin(C) = $\frac{AB}{AC}$

All other ratios are correct

18.

Find the median and mode of this set of data

5 , 1, 3, 5, 8 , 10, 8, 14

The mode is the value you see more often. 5 and 8 are seen more often, so they are the modes

of this set of data

The median is the value is the middle after the set is put in order from least to greatest or from greatest to least.

1 3 5 5 8 8 10 14

1 3 5 5 middle value 8 8 10 14

The middle value is between 5 and 8

What number is between 5 and 8 ? It is between 6 and 7 as shown below.

5 6 middle value 7 8

What is between 6 and 7 ? It is 6.5.

The median is 6.5

To get the median, you could also take the average of 5 and 8

median = $\frac{5+8}{2}$ = $\frac{13}{2}$ = 6.5

19.

Find the equations that are parallel and perpendicular to x + 5y = 50 and passing through the points (5, 10)

First, put x + 5y = 50 in stantard from

x + 5y = 50

x – x + 5y = 50 – x (Minus x from both sides)

5y = 50 – x

5y = -x + 50

$\frac{5}{5}y = \frac{-x}{5} + \frac{50}{5}$ (Divide both sides by 5)

1y = $\frac{-x}{5}$ + 10

y = $\frac{-1x}{5}$ + 10 (Standard form)

y = $\frac{-1}{5}x$ + 10

Let us look for the equation that is parallel to y = $\frac{-1}{5}x$ + 10

When two equations are parallel, they have the same slope.

Therefore, the slope of the equation that is parallel to y = $\frac{-1}{5}x$ + 10 is equal to $\frac{-1}{5}$

We get y = $\frac{-1}{5}x$ + b

All we need to do now is to find b

Use (5, 10) to find b. x= 5 and y = 10. Just replace these values into the equation

$10 = \frac{-1}{5} \times 5 + b$

$10 = \frac{-1}{5} \times \frac{5}{1} + b$

$10 = \frac{-5}{5} + b$

10 = -1 + b

10 + 1 = -1 + 1 + b

11 = b

$y = \frac{-1}{5}x + 11$

The equation that is parallel to $y = \frac{-1}{5}x + 11$ and passing through (5, 10) is

$y = \frac{-1}{5}x + 11$

Let us look for the equation that is perpendicular to $y = \frac{-1}{5}x + 10$

When two equations are perpendicular, the product of their slopes is equal to -1.

Let m be the slope of the equation that is perpendicular to $y = \frac{-1}{5}x + 10$

$m \times \frac{-1}{5} = -1$

Since $\frac{5}{1} \times \frac{-1}{5} = \frac{-5}{5} = -1$, $m = \frac{5}{1} = 5$

We get y = 5x + b

All we need to do now is to find b

Use (5, 10) to find b.   x= 5 and y = 10. Just replace these values into the equation

10 = 5 × 5 + b

10 = 25 + b

10 – 25 = 25 – 25 + b

-15 = b

y = 5x + -15

The equation that is perpendicular to   y = $\frac{-1}{5}x$ + 11 and passing through (5, 10) is y = 5x + - 15

20.

Simplify (1.5 × 10^{8}) (6 × 10^{-8})

Rewrite the expression

(1.5 × 10^{8}) (6 × 10^{-8}) =   1.5 × 10^{8} × 6 × 10^{-8} (Remove parentheses)

= 1.5 × 6 × 10^{8} × 10^{-8}

= 9 × 10^{8} × 10^{-8}

= 9 × 10^{8+-8}

= 9 × 10^{0}

= 9 × 1

= 9

21. Simplify each radical expression

a. 5 $\sqrt{32}$ – 3 $\sqrt{2}$

5 $\sqrt{32}$ – 3 $\sqrt{2}$ = 5 $\sqrt{16\times 2}$ – 3 $\sqrt{2}$

$= 5 \quad (\sqrt{16} \times \sqrt{2})$ - 3 $\sqrt{2}$

$= 5 \quad \sqrt{16} \times \sqrt{2} - 3 \sqrt{2}$

$= 5 \times 4 \times \sqrt{2} - 3 \sqrt{2}$

$= 20 \sqrt{2} - 3 \sqrt{2}$

$= \sqrt{2} \times (20 - 3)$

$= \sqrt{2} \ (17)$

$= 17 \sqrt{2}$

b.

$$\sqrt{\frac{8x^8}{2x^2}}$$

$$\sqrt{\frac{8x^8}{2x^2}} = \sqrt{\frac{8}{2} \times \frac{x^8}{x^2}}$$

$$= \sqrt{4 \times \frac{x^8}{x^2}}$$

$$= \sqrt{4 \times x^8 \times x^{-2}}$$

$$= \sqrt{4 \times x^{8+-2}}$$

$$= \sqrt{4 \times x^6}$$

$= \sqrt{4} \times \sqrt{x^6}$

$= 2 \times x^3$

22. Solve the inequality 2y - 4x < 6 for y. Then, give 2 points that are solutions.

2y – 4x < 6

2y – 4x + 4x < 6 + 4x (Add 4x to both sides of the equation)

2y < 6 + 4x

$\frac{2}{2}y < \frac{6}{2} + \frac{4}{2}x$

1y < 3 + 2x

y < 2x + 3

To find points that are solutions, set y equal to 2x + 3 and graph the line y = 2x + 3

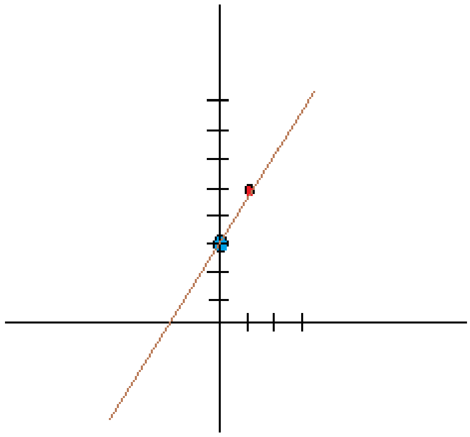
When looking at the equation 2x + 3, 3 is the y-intercept and it goes on the y-axis

The dot in blue represents 3.

The number 2 next to the x is the slope. 2 = $\frac{2}{1}$

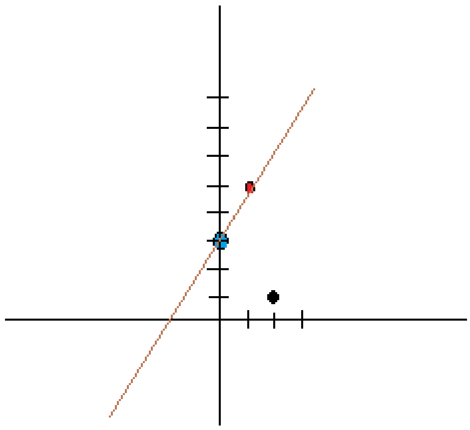
Starting where the blue dot is, $\frac{2}{1}$ means to go up 2 units. Then, go to the right 1 unit.

We end up where the red dot is.



To see where the solutions are, we need to perform a test.

Test: Pick any point you like either on the right or left side of the line.



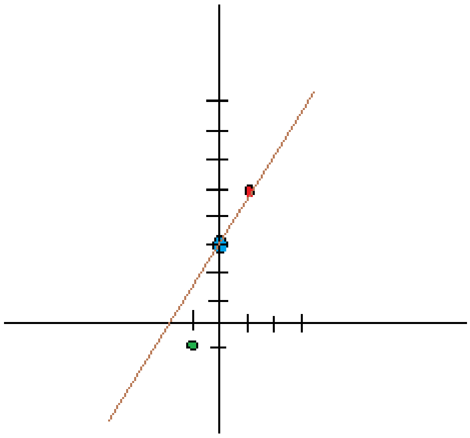
Pick (2, 1) for instance on the right side. We show this with a black dot.

Verify that it may work by replacing the x and the y-values into $2y - 4x < 6$

$2 \times 1 - 4 \times 2 = 2 - 8 = -6$ and $-6 < 6$. **It works**!

As a rule of thumb, if it works for just 1 point on the right side, it is going to work for any other point chosen on that side.

Pick (-1, -1) still on the right side of the line with a green dot.



$2 \times -1 - 4 \times -1 = -2 + 4 = 2$ and $2 < 6$. **It works**!

Therefore, 2 points that are solutions of this inequality are (2, 1) and (-1, -1)

23.

a. Factor x^2 - 8x - 20

b. Solve x^2 - 8x – 20

a. Factor x^2 - 8x – 20

$x^2 - 8x - 20 = (x_1 + \text{blank}) \times (x_2 + \text{blank})$

To find blanks, look for factors of -20 that add up to -8

-20 = 20 × -1

= -20 × 1

= 5 × -4

= -5 × 4

= 10 × -2

= -10 × 2

The pair of factors that add up to -8 is -10 and 2 since 2 + -10 = -8

Replace the two blanks with -10 and 2

x^2 - 8x – 20 = $(x_1 + -10) \times (x_2 + 2)$

b. Solve x^2 - 8x – 20

The meaning of solve is to find values of x that will make x^2 - 8x – 20 equal to zero.

$x^2 - 8x - 20 = 0$

$(x_1 + -10) \times (x_2 + 2)$ is zero if $(x_1 + -10) = 0$ or $(x_2 + 2) = 0$

$x_1 + -10 = 0$

$x_1 + -10 + 10 = 0 + 10$

$x_1 = 10$

$x_2 + 2 = 0$

$x_2 + 2 - 2 = 0 - 2$

$x_2 = -2$

24.

Solve $12x^2 + 7x - 10$

$12x^2 + 7x - 10 = 0$

Factor $12x^2 + 7x - 10$. We will factor by **grouping**

$12x^2 + 7x - 10 = 12x^2 + -15x + 8x - 10$ $(7x = -15x + 8x)$

$= (12x^2 + -15x) + (8x - 10)$ Group

$= 3x(4x - 5) + 2(4x - 5)$ 4x – 5 is a common factor, so use it to factor

$= (4x - 5)(3x + 2)$

$12x^2 + 7x - 10 = (4x - 5)(3x + 2) = 0$

$(4x + -5) \times (3x + 2)$ is zero if $(4x + -5) = 0$ or $(3x + 2) = 0$

$4x + -5 = 0$

$4x + -5 + 5 = 0 + 5$

$4x = 5$

$$\frac{4}{4}x = \frac{5}{4}$$

$$1x = \frac{5}{4}$$

$$x = \frac{5}{4}$$

$3x + 2 = 0$

$3x + 2 - 2 = 0 - 2$

$3x = -2$

$$\frac{3}{3}x = \frac{-2}{3}$$

$$1x = \frac{-2}{3}$$

$x = \frac{-2}{3}$

25.

Solve the following proportion by setting up a proportion

The hours of operation of a park is 8:00 am - 10:00 pm Wednesday through Sunday.
A Ferris wheel can make a 360 degrees turn 5 times every 10 minutes.
How many 360 degrees turn can the Ferris wheel make in a week?

The park is open from 8:00 am to 10:00 pm; That is 14 hours.

The park is opened Monday through Sunday; That is 5 days.

Number of hours the park is opened is 14 × 5 = 70 hours.

Model the situation

5 rotations ____________________ 10 minutes

Number of rotations (every 60 minutes) ____________________ 60 minutes

Here is the **proportion**

$$\frac{5\,rotations}{number\ of\ rotations} = \frac{10\,minutes}{60\,minutes}$$

Cross multiply

5 rotations × 60 minutes = number of rotations × 10 minutes

300 rotations × minutes = number of rotations × 10 minutes

number of rotations × 10 minutes = 300 rotations × minutes

number of rotations × $\frac{10\,minutes}{10\,minutes}$ = $\frac{300\,rotations \times minutes}{10\,minutes}$

number of rotations × $\frac{10\,\cancel{minutes}}{10\,\cancel{minutes}}$ = $\frac{300\,rotations \times \cancel{minutes}}{10\,\cancel{minutes}}$

number of rotations × 1 = $\frac{300\,rotations}{10}$

number of rotations = 30 rotations in an hour

Model the situation again

30 rotations ____________________ 1 hour

Number of rotations ____________________ 70 hours

Here is the proportion

$$\frac{30\,rotations}{number\ of\ rotations} = \frac{1\,hour}{70\,hours}$$

Cross multiply

30 rotations × 70 hours = number of rotations × 1 hour

number of rotations × 1 hour = 30 rotations × 70 hours

number of rotations × 1 hour = 2100 rotations × hour number of rotations × $\frac{1\,hour}{1\,hour}$ = $\frac{2100\,rotations \times hours}{1\,hour}$

number of rotations × $\frac{1\,\cancel{hour}}{1\,\cancel{hour}}$ = $\frac{2100\,rotations \times \cancel{hours}}{1\,\cancel{hour}}$

number of rotations × 1 = 2100 rotations

number of rotations = 2100 rotations

26.

How can you find out how many solutions a quadratic equation has without solving the quadratic equation?

The standard form of a quadratic equation is ax^2 + bx + c = 0

Just compute b^2 - 4ac

If b^2 - 4ac = 0, the quadratic equation has only 1 solution

If b^2 - 4ac > 0, the quadratic equation has only 2 solutions

If b^2 - 4ac < 0, the quadratic equation has complex solutions (You don't need to worry about this one now)

Where did b^2 - 4ac come from?

It came from the quadratic formula. x = $\frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

It is called the radicand.

27.

The solution of the system ax - 4y = 8 and 6x + by = -30 is (-2, -3)

1. Find a and b

2. Replace a and b into the system. Then solve the system to verify that the solution is indeed (-2, -3)

1. To find a and b, use (-2, -3)

Replace x = -2 and y = -3 into each equation ax - 4y = 8 and 6x + by = -30

a×-2 - 4 × -3 = 8

-2a + 12 = 8

-2a + 12 – 12 = 8 – 12

-2a = -4

$$\frac{-2}{-2}a = \frac{-4}{-2}$$

1a = 2

a = 2

6x + by = -30

6 × -2 + b × -3 = -30

-12 + -3b = -30

-12 + 12 + -3b = -30 + 12

-3b = -18

$$\frac{-3}{-3}b = \frac{-18}{-3}$$

1b = 6

b = 6

2.

2x - 4y = 8 equation 1

6x + 6y = -30 equation 2

Multiply equation 1 by -3. Keep equation 2 the same

-3 (2x – 4y) = -3 × 8

6x + 6y = -30

-6x + 12y = -24 equation 3

6x + 6y = -30 equation 4

Add equation 3 and equation 4

-6x +	12y	=	-24
6x +	6y	=	-30
0	18y	=	-54

18y = -54

$$\frac{18}{18}y = \frac{-54}{18}$$

1y = -3

y = -3

Replace y into equation 3 or equation 4 to get x

6x + 6 × -3 = -30

6x + -18 = -30

6x + -18 + 18 = -30 + 18

6x = -12

$$\frac{6}{6}x = \frac{-12}{6}$$

1x = -2

x = -2

28.

Find the value of $_6P_4$

$$_6P_4 = \frac{6!}{(6-4)!}$$

$$= \frac{6!}{(2)!}$$

$$= \frac{6 \times 5 \times 4 \times 3 \times 2!}{2!}$$

$$= \frac{6 \times 5 \times 4 \times 3 \times \cancel{2}!}{\cancel{2}!}$$

= 6 × 5 × 4 × 3

= 30 × 4 × 3

= 120 × 3

= 360

29.

Simplify $\frac{30x^4y^8z}{50x^{10}y^7z}$

$$\frac{30x^4y^8z}{50x^{10}y^7z} = \frac{3 \times 10x^4 \times y^7 \times y \times z}{5 \times 10x^4 \times x^6 \times y^7 \times z}$$

Anything you see on top and at the bottom will cancel.

We show these with the same color below

$$\frac{30\,x^4\,y^8\,z}{50\,x^{10}\,y^7\,z} = \frac{3\times 10\,x^4\times y^7\times y\times z}{5\times 10\,x^4\times x^6\times y^7\times z}$$

$$= \frac{3y}{5x^6}$$

30.

What is the value of the f(x) = $\frac{-8}{x--2}$ for x = -2

f(-2) = $\frac{-8}{2--2}$ = $\frac{-8}{2+2}$ = $\frac{-8}{4}$ = -2

31.

Solve the equation -2 + $\sqrt{y-5}$ = 10

-2 + $\sqrt{y-5}$ = 10

-2 + 2 $\sqrt{y-5}$ = 10 + 2 (Add 2 to both sides)

$\sqrt{y-5}$ = 12

$[\sqrt{(y-5)}]^2$ = 12^2 (Square both sides) { $[\sqrt{(anything)}]^2 = anything$

y – 5 = 144

y – 5   + 5   = 144 + 5

y = 149

32.

Write a function rule for the following table

x	y
0	-4
6	-5
12	-6
24	-7

We are looking for y = mx + b

m is the slope and b is the y-intercept

First, find the slope using 2 points. We can use (0, -4) and (6, -5)

$$m=\frac{y_1-y_2}{x_1-x_2}$$

$$m=\frac{-4--5}{0-6}=\frac{-4+5}{-6}=\frac{1}{-6}=\frac{-1}{6}$$

m = $\frac{-1}{6}$

y = mx + b = $\frac{-1}{6}x$ + b

When x = 0, y = -4, so y = -4 is the y-intercept or b

y = $\frac{-1}{6}x$ + -4

f(x) = $\frac{-1}{6}x$ + -4

33. Suppose x - 25 = -75, what is x + 25? What is x + 50 ?

x – 25 = - 75

x – 25 + 25 = -75 + 25 (Add 25 to both sides of the equation)

x + 0 = -50

x + 25 = -50 + 25 (Add 25 to both sides of the equation)

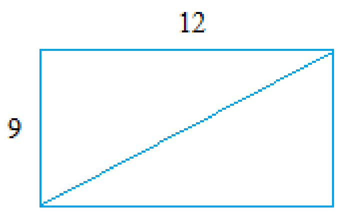
x + 25 = -25

x + 25 + 25 = -25 + 25 (Add 25 to both sides of the equation)

x + 50 = 0

34.

What is the length of the diagonal of a rectangle with sides equal to 9 and 12 ?



Use the Pythagorean Theorem

$c^2 = a^2 + b^2$

$c^2 = 9^2 + 12^2$

$c^2 = 81 + 144$

$c^2 = 225$

c = $\sqrt{(225)}$

c = 15

35.

For some equations, the "solution" turns out to be **not** a solution of the equation you are solving, this solution is called an **extraneous solution**.
Equations that have absolute value and square root sign can give extraneous solutions

36. Solve and graph $4|2x - 5| \geq 20$

$4|2x - 5| \geq 20$

$\frac{4}{4}|2x - 5| \geq \frac{20}{4}$ (Divide both sides by 4)

$|2x - 5| \geq 5$

We get two equations to solve $2x - 5 \geq 5$ and $2x - 5 \leq -5$

$2x - 5 \geq 5$
$2x - 5 + 5 \geq 5 + 5$
$2x \geq 10$
$\frac{2}{2}x \geq \frac{10}{2}$

$x \geq 5$

$2x - 5 \leq -5$

2x – 5 + 5 ≤ -5 + 5

2x ≤ 0

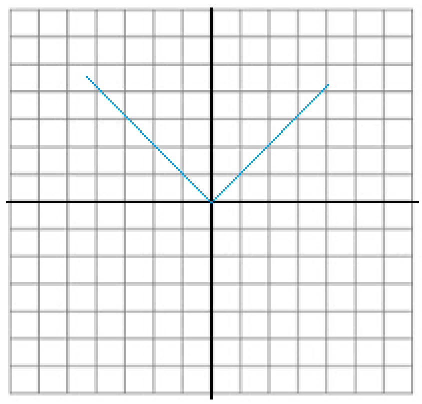
$$\frac{2}{2}x \le \frac{0}{2}$$

x ≤ 0

The graph is shown below

37.

The graph of y = |x| is shown below.



The meaning of y =|x| + b is to move the graph of | x | b units up

The meaning of y =|x| - b is to move the graph of | x | b units down

The meaning of y =|x - a| is to move the graph of | x | a units to the right

The meaning of y =|x + a| is to move the graph of | x | a units to the left

The meaning of y = a|x| is to make the graph of |x| more narrow or more wide than the graph of y = |x|

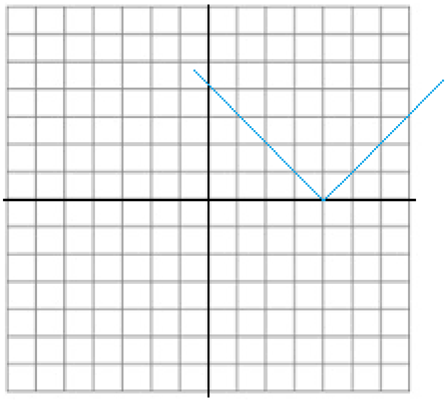
If a is bigger than 1, the graph will be more narrow.

If a is smaller than 1, but bigger than 0, the graph will be more wide

a. $y = |x - 4|$

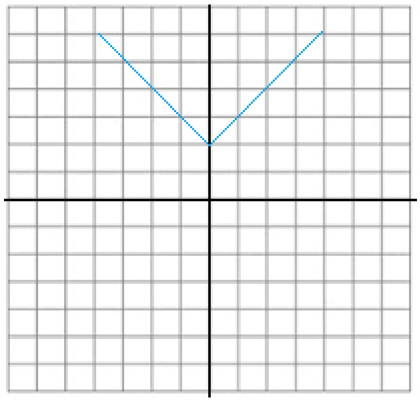
The meaning of $y = |x - 4|$ is to move the graph of $|x|$ 4 units to the right

As long as you don't change the shape of the new graph as you move it to the right, you should be fine. The lines of the new graph should be parallel to the old one.



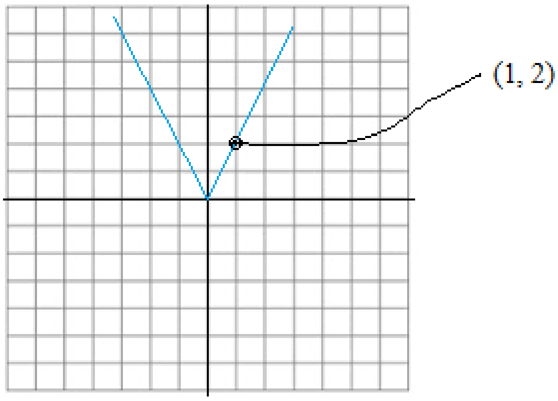
b. $y = |x| + 2$

The meaning of $y = |x| + b$ is to move the graph of $|x|$ 2 units up



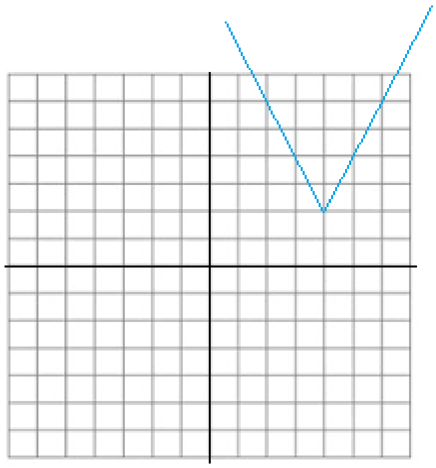
c. $y = 2|x|$

To graph $y = 2|x|$, make sure the line passes through (1, 2) instead of (1,1)
If you were graphing $y = 3|x|$, you will make sure that the graph passes through (1, 3) instead of (1, 1), and so forth...



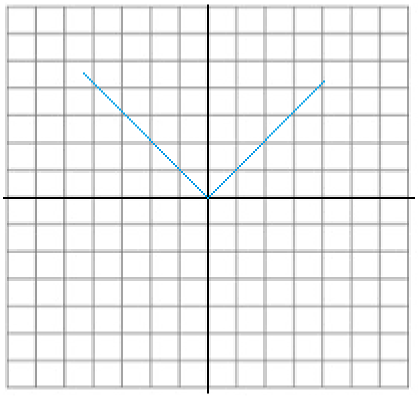
d. $y = 2|x - 4| + 2$

The 2 on the left of $|x – 4|$ means that it will have the same shape as $y = 2|x|$ and we already graphed that above.
Therefore, take that shape and move it 4 units to the right and then 2 units up and you are done.

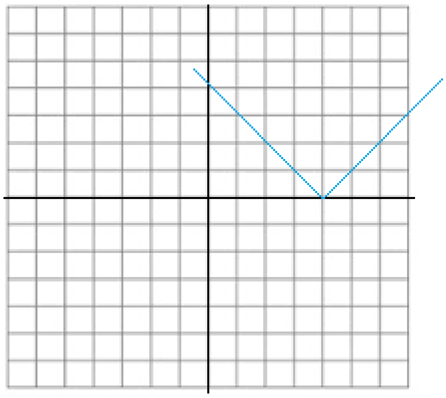


We show all the graphs below so you can see them all together and better visualize what is going on with them.

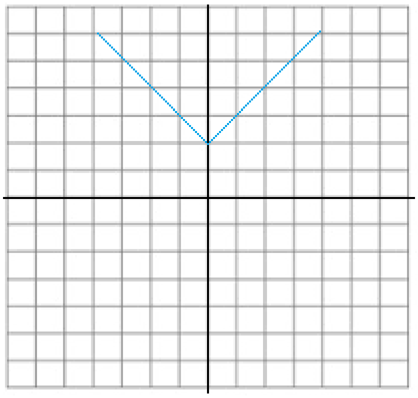
$y = |x|$



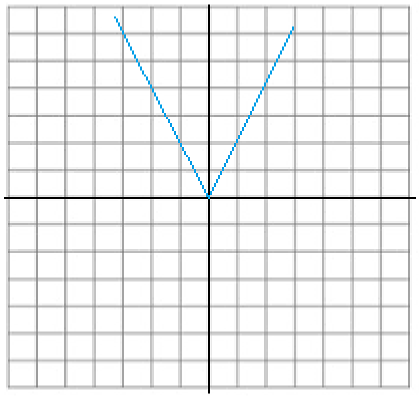
$y = |x - 4|$



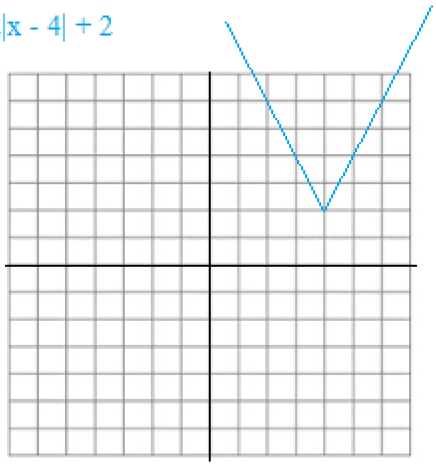
$y = |x| + 2$



$y = 2|x|$



$y = 2|x - 4| + 2$



38.

The product of two positive integers is 45. The first is 4 less than the second. Find the integers by setting up an equation.

Let x be the first integer. Let y be the second integer.

The first is 4 less than the second. Mathematically, $x = y - 4$

The product of two positive integers is 45. Mathematically, $y \times (y - 4) = 45$

So what you need to do is to solve $y \times (y - 4) = 45$

$y \times (y - 4) = 45$

$y^2 - 4x = 45$

$y^2 - 4x - 45 = 45 - 45$

$y^2 - 4x - 45 = 0$

$y^2 - 4x - 45 = (y_1 + \text{blank}) \times (y_2 + \text{blank}) = 0$

To find blanks, look for factors of -45 that add up to -4

$-45 = -9 \times 5$ and $-9 + 5 = -4$

$y^2 - 4x - 45 = (y_1 + -9) \times (y_2 + 4) = 0$

Solve $(y_1 + -9) = 0$ and $(y_2 + 4) = 0$

$y_1 + -9 = 0$

$y_1 + -9 + 9 = 0 + 9$

$y_1 = 9$

When $y = 9$, $x = y - 4 = 9 - 4 = 5$

9 and 5 are the positive integers we are looking for.

Indeed 9 and 5 are positive and 9 times 5 = 45.

When y = -4, x = -4 – 4 = -8. These don't solve the problem, so -4 is an extraneous solution

39.

What is the percent of decrease from 12 to 4?

Percent of decrease is $\frac{12-4}{12}$ = $\frac{8}{12}$ = $\frac{2}{3}$ = 0.66666 = 66.66%

40.

A company enrolls computer programmers and secretaries at a ratio of 5 to 3. There are a total of 24 secretaries and computer programmers. How many people are secretaries? How many are computer programmers?

Let x be the number of computer programmers.
Let y be the number of secretaries.

$$\frac{x}{y}=\frac{5}{3}$$

x + y = 24

Using $\frac{x}{y}=\frac{5}{3}$, cross multiply to get 3x = 5y

3x = 5y is the same as 3x – 5y = 0

You have 2 equations to solve

x + y = 24 equation 1

$3x – 5y = 0$ equation 2

Mulliply equation 1 by -3 and leave equation 2 as it is.

$-3 (x + y) = -3 \times 24$

$3x – 5y = 0$

$-3x + -3y = - 72$ equation 3

$3x – 5y = 0$ equation 4

Add equation 3 to equation 4

$$\begin{array}{cccc} -3x + & -3y & = & -72 \\ 3x & -5y & = & 0 \\ \hline 0 & -8y & = & -72 \end{array}$$

$-8y = -72$

$$\frac{-8}{-8}y = \frac{-72}{-8}$$

$1y = 9$

$y = 9$

Use $x + y = 24$ to get x

$x + 9 = 24$

$x + 9 - 9 = 24 – 9$

$x = 15$

There are 15 computer programmers and 9 secretaries

41.

The price p of an item is sold with a discount of 20%. The expression that represents the sale price of the item is ____________

A. p - 20p   B. 0.20p   C. p + 0.20p   D. p - 0.20p

Discount $= p \times 20\%$

$= p \times \frac{20}{100}$

$= p \times 0.20$

$= 0.20p$

Sale price = price of item – discount

$= p - 0.20p$

42.

$y > \frac{-2}{3}x + 1$

To find points that are solutions, set y equal to $\frac{-2}{3}x + 1$ and graph the line

$y = \frac{-2}{3}x + 1$

When looking at the equation $\frac{-2}{3}x + 1$, 1 is the y-intercept and it goes on the y-axis

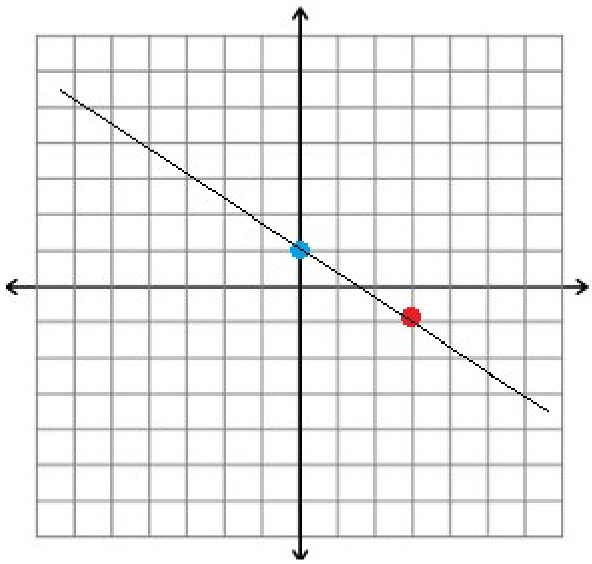
The dot in blue represents 1.

The number $\frac{-2}{3}$ next to the x is the slope.

Starting where the blue dot is, $\frac{-2}{3}$ means to go down 2 units.

Then, go to the right 3 unit.

We end up where the red dot is. Finally, make the line



Pick (3, 1) for instance above the line. We show this with a black dot.

Verify that it may work by replacing the x and the y-values into $y > \frac{-2}{3}x + 1$

Is $1 > \frac{-2}{3} \times 3 + 1$?

Is $1 > \frac{-2}{3} \times \frac{3}{1} + 1$

Is $1 > \frac{-6}{3} + 1$

Is 1 > -2 + 1

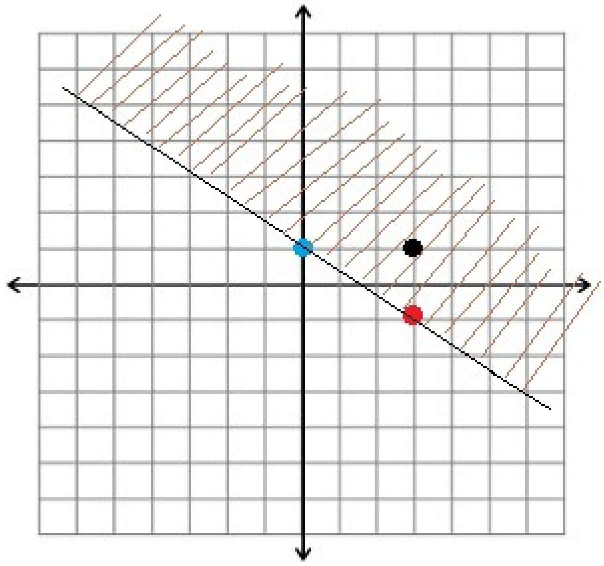
Is 1 > -1

Since 1 is bigger than -1, the point (3,1) is a solution.

As a rule of thumb, if it works for just 1 point above the line , it is going to work for any

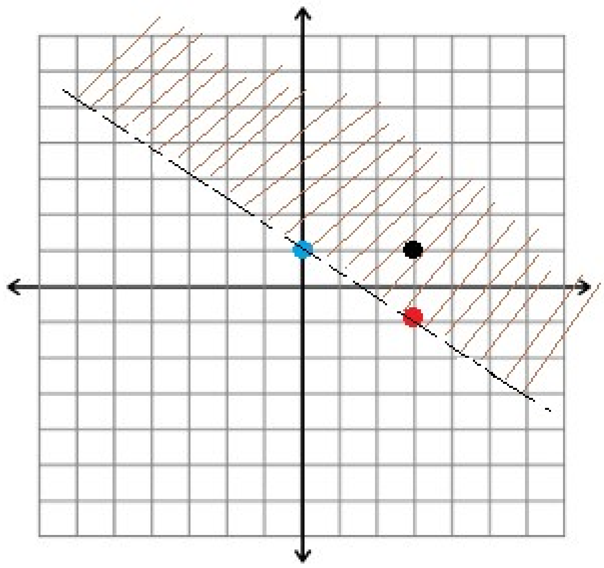
other point chosen on that side. We show all the solutions by shading above the line with

brown lines.



The black line is the equation y = $\frac{-2}{3}x$ + 1 . However, we are solving y > $\frac{-2}{3}x$ + 1

So any point on the black line is **not** included in the solution. We show this by making the line broken.



43.

The sides of a rectangle are -5x + 10 and x - 4

a. Write an expression for the perimeter.
b. Write an expression for the area.

c. For what value of x is the area the biggest?

a. Perimeter = -5x + 10 + -5x + 10 + x – 4 + x – 4

= -5x + -5x + x + x + 10 + 10 – 4 – 4

= - 10x + 2x + 20 – 8

= - 8x + 12

b. Area = (-5x + 10) × (x – 4)

= -5x × x + -5x × - 4 + 10 × x + 10 × - 4

= $-5x^2$ + 20x + 10x + -40

= $-5x^2$ + 30x + -40

c. This is the same problem as question 10 above. Here though, we are looking for a maximum.

As we did in exercise 10, solve (-5x + 10) × (x – 4) = 0

-5x + 10 = 0

-5x + 10 – 10 = 0 – 10

-5x = -10

$$\frac{-5}{-5}x = \frac{-10}{-5}$$

x = 2

x - 4 = 0

x - 4 + 4 = 0 + 4

x = 4

Since the x-value of the maximum is between x_1 and x_2, Just find out what number is between 2 and 4

Look at this list: 2, 3, 4

The middle number is between 2 and 4. That number is 3

Maximum area = (-5x + 10) × (x – 4)

= (-5 × 3 + 10) × (3 – 4)

= (-15 + 10) × (-1)

= (-5) (-1)

= 5

44.

Simplify $\frac{4x - 20}{x^2 - 10x + 25}$

$$\frac{4x - 20}{x^2 - 10x + 25}$$

Factor 4x – 20.

4x – 20 = 4 (x – 5)

x^2 – 10x + 25 = (x + blank) × (x + blank)

Look for factors of 25 that add up to -10

25 = - 5 × -5 and -5 + -5 = -10

Fill in the blank with – 5 and -5

x^2 – 10x + 25 = (x + -5) × (x + -5)

$$\frac{4x - 20}{x^2 - 10x + 25} = \frac{4(x-5)}{(x-5)\times(x-5)} = \frac{4(x-5)}{(x-5)\times(x-5)} = \frac{4}{x-5}$$

Notice that the area in red gets cancelled.

45.

Find the horizontal and vertical asymptotes of the function below

f(x) = $\frac{-10}{x-3}$ + 5

Vertical asymptotes are found for those values where the denominator is not defined

When x = 3, x – 3 = 3 – 3 = 0

-10 over 0 is not defined, so x = 3 is a vertical asymptote

To find horizontal asymptotes, make x very big and see what y or f(x) is going to get close to

When x = 10000, f(x) = $\frac{-10}{10000-3}$ + 5 = -0.001 + 5

When x = 1000000, f(x) = $\frac{-10}{1000000-3}$ + 5 = -0.00001 + 5

As x keeps getting bigger and bigger, $\frac{-10}{x-3}$ is going to be so small, it will be as if it did not exist.

The horizontal asymptote is therefore 5

46.

Simplify $\frac{x-6}{x^2-x}$ ÷ $\frac{x^2-4x-12}{3x^2-3x}$

x^2 – x = x(x – 1)

$3x^2$ – 3x = 3x (x – 1)

x^2 – 4x – 12 = (x – 6) (x + 2)

Note that as we already did before -6 and 3 are just factors of -12 that add up -4

$$\frac{x-6}{x^2-x} \div \frac{x^2-4x-12}{3x^2-3x}$$

$$\frac{x-6}{x^2-x} \div \frac{x^2-4x-12}{3x^2-3x} = \frac{x-6}{x^2-x} \times \frac{3x^2-3x}{x^2-4x-12}$$

$$= \frac{x-6}{x(x-1)} \times \frac{3x(x-1)}{(x-6)\times(x+2)}$$

The factor in blue on top gets cancelled with the factor in blue at the bottom.

The factor in red on top gets cancelled with the factor in red at the bottom.

The factor in green on top gets cancelled with the factor in green at the bottom.

$$\frac{x-6}{x^2-x} \div \frac{x^2-4x-12}{3x^2-3x} = \frac{3}{x+2}$$

47.

Graph the following system of equations?

2y > x + 4
3y + 3x >13

Solve 2y > x + 4

2y > x + 4

$$\frac{2}{2}y > \frac{x}{2} + \frac{4}{2}$$

$$y > \frac{x}{2} + 2$$

3y + 3x > 13

3y + 3x – 3x > 13 – 3x

3y > -3x + 13

$\frac{3}{3}y$ > $\frac{-3}{3}x$ + $\frac{13}{3}$

y > -1x + $\frac{13}{3}$ or y > -1x + 4 $\frac{1}{3}$

Graph y > $\frac{x}{2}$ + 2 and y > -1x + 4 $\frac{1}{3}$

Graph y > $\frac{1}{2}x$ + 2

First graph y = $\frac{1}{2}x$ + 2

The y-intercept is 2. We show this with the first blue dot on the y-axis

The slope is 1 over 2. Starting from the first blue dot, this means to go up 1 unit and to the right 2 units. We end up where you see the second blue dot. Finally, draw the line.

Pick (4, 8) for instance above the line. We show this with a red dot.

Verify that it may work by replacing the x and the y-values into y > $\frac{1}{2}x$ + 2

Is 8 > $\frac{1}{2}$ × 4 + 2 ?

Is 8 > $\frac{1}{2}$ × $\frac{4}{1}$ + 2

Is 8 > $\frac{4}{2}$ + 1

Is 8 > 2 + 1

Is 8 > 3

Since 8 is bigger than 3, the point (4, 8) is a solution.

As a rule of thumb, if it works for just 1 point above the line , it is going to work for any other point chosen on that side. We show all the solutions by shading above the line with brown lines.

Then, graph y = $y > -1x + 4\frac{1}{3}$

The y-intercept is $4\frac{1}{3}$. We show this with the first green dot on the y-axis

The slope is -1 = $\frac{-1}{1}$.

Starting from the first green dot, this means to go down 1 unit and to the right 1 unit. We end up where you see the second green dot. Finally, draw the line.

Pick (6, 1) for instance above the line. We show this with a black dot.

Verify that it may work by replacing the x and the y-values into $y > -1x + 4\frac{1}{3}$

Is $1 > -1 \times 6 + 4\frac{1}{3}$?

Is $1 > -2\frac{1}{3}$

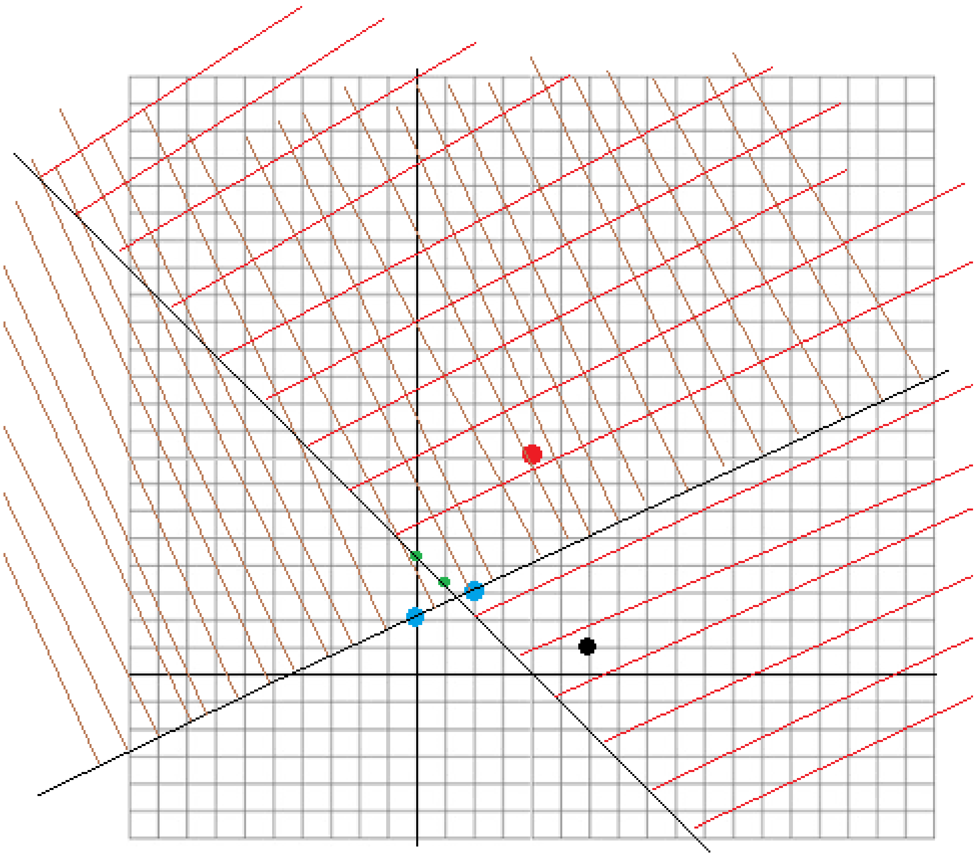
Is $1 > -2 + \frac{1}{3}$

Is $1 > -1\frac{1}{3}$

Is $1 > \frac{-4}{3}$

Since 1 is bigger than $\frac{-4}{3}$, the point (6, 1) is a solution.

As a rule of thumb, if it works for just 1 point above the line , it is going to work for any other point chosen on that side. We show all the solutions by shading above the line with red lines.



48.

Find the LCD of $x^5 y$ and $x^2 y^4$

For each variable, the LCD is the expression with the highest exponent

For the variable x, the expression with the highest exponent is x^5

For the variable y, the expression with the highest exponent is y^4

The LCD is $x^5 y^4$

49.

a. How many different ways can you choose 3 books from the 8 books on your shelf?

b. A 20 member club needs 2 members to lead . How many different pairs are possible?

a. For the first book, you have 8 choices

For the second book, you have 7 choices

For the third book, you have 6 choices

Number of different ways = 8 × 7 × 6 = 56 × 6 = 336

You could also use the formula

$$_{8}P_{3} = \frac{8!}{(8-3)!}$$

$$= \frac{8!}{(5)!}$$

$$= \frac{8 \times 7 \times 6 \times 5!}{5!}$$

$$= \frac{8 \times 7 \times 6 \times \cancel{5}!}{\cancel{5}!}$$

= 8 × 7 × 6 = 336

b. Here, order does not matter since the members just neeed two people to lead.

$$_{20}C_{2} = \frac{20!}{2!(20-2)!}$$

$$= \frac{20!}{2!(18)!}$$

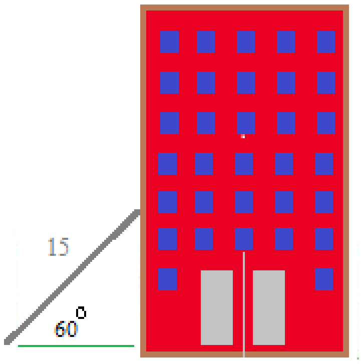
$$= \frac{20 \times 19 \times 18!}{2!18!}$$

$$= \frac{20 \times 19 \times \cancel{18}!}{2!\cancel{18!}}$$

$$= \frac{20 \times 19}{2}$$

= 10 × 19 = 190

50. John has a 15 feet ladder. When he leaned the ladder against a building, the ladder makes an angle of 60 degrees with the ground. What is the distance from the side of the building to the base of the ladder?



Let x be the distance from side of the building to the base of the latter (Shown in green)

Cos(60 degrees) $= \dfrac{x}{15}$

$$\frac{1}{2} = \frac{x}{15}$$

Cross multiply: $1 \times 15 = x \times 2$
$15 = 2x$

Since 15 = 2 times 7.5. x = 7.5

The distance from the side of the building to the base of the ladder is 7.5